Pricing Investor Impact

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ABSTRACT

This paper presents an asset pricing model where demand shifts by consequentialist investors influence firms' production choices. If the potential to influence production is correctly reflected in an investor's demand, further demand shifts should not improve their utility. Using this principle, a theoretical valuation formula for investor impact is derived. Calibration on US stocks suggests a $1 increase in investor demand generates 3 cents in productive assets. This implies significant potential for investor impact, yet requires extreme 'prices of impact' to explain observed sustainability-related portfolio tilts. If investors willingness to pay high prices for impact was consistent across opportunities, much more would be allocated to asset classes other than public equities. The model offers a general framework for quantitatively answering questions related to both universal ownership and impact investing.

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In sustainable investment, "investor impact" refers to the change in sustainability-related outcomes generated by an investor's actions, encompassing both social and environmental outcomes (Brest et al., 2018; Kölbel et al., 2020; Busch et al., 2021). Assessing these changes relative to counterfactual scenarios presents a formidable challenge. However, prominent impact-oriented investors, such as development finance institutions, are routinely expected to demonstrate their "additionality"—that is, to provide evidence that their investments generate investor impact (Carter et al., 2021). Underscoring the importance of such assessments, a recent consensus of over 3,000 practitioner organizations recognized "investor contribution" (a synonym for investor impact) as a key dimension for impact management (Boiardi and Stout, 2021; Impact Frontiers, 2023).

Answering Starks (2023)'s call for more theoretical frameworks that consider the differences between investors with different nonpecuniary preferences or "values", this paper presents an asset pricing model that allows investors to have "impact preferences", with an explicit concern for aggregate sustainability-related outcomes. This contrasts with "holdings-based preferences" that depend only on the characteristics of the assets held by an investor[1]. I show that investors with impact preferences develop endogenous portfolio tilts that can be dramatically different from the exogenous tilts emphasized in the literature on holdings-based preferences. These "impact tilts" are shaped not only by exogenous nonpecuniary characteristics, but as much or more by firm and market characteristics that determine the potential for investor impact.

This paper addresses three central questions: First, how can investor impact be formally defined within an asset pricing model? Second, what is the optimal policy for an investor with impact preferences? Third, what effects on asset prices, sustainability-related outcomes, and investor utility should be expected from investors pursuing these policies?

To answer these questions, this paper develops an asset pricing model based on a supply and demand system with a sustainability-related good. The model builds on insights from papers across the literature, including Koijen and Yogo (2019), Van der Beck (2022), Betermier et al. (2022), Pástor et al. (2021), and Merton (1987). It features many heterogeneous firms and agents, yet to first order it is highly tractable and yields intuitive expressions.

[1]See Green and Roth (2021) and Edmans and Kacperczyk (2022) for further discussion. In the terminology of Yasuda (2023), impact preferences are consequentialist while holdings-based preferences are deontological (Kantian). Heinkel et al. (2001) and Pástor et al. (2021) are prominent examples of models with holdings-based preferences. In general, such preferences can include any utility that depends on asset holdings, including due to subjective financial beliefs. In the context of sustainable investing, they are also known as "values-aligned" or "warm glow" preferences in reference to the work of Andreoni (1990).

The primary channel for investor impact that I consider involves changes in investor demand affecting the scale of firms' production. Firms differ in their "social productivity", which refers to the amount that marginal increases in their production contributes to the sustainability-related good. The degree to which a shift in demand leads to a corresponding shift in supply (and thus, production of the sustainability-related good) is captured by "contribution multipliers" that depend on the supply and demand elasticities associated with each asset.

To first order, the contribution multiplier $\mathcal{C}$, which defines the ratio between a marginal investor's demand shift (ΔK_i) and the resulting net shift in supply of the target firm (ΔK), is given by

$$\mathcal{C} \approx \frac{\Delta K}{\Delta K_i} \approx \frac{\zeta_S}{\zeta_D + \zeta_S}, \tag{1}$$

where ζ_D represents the price elasticity of demand and ζ_S denotes the price elasticity of supply. For example, for a stock with $\mathcal{C} = 0.03$, a shift in ΔK_i that correspond to a \$1 increase in demand can be expected to generate ΔK that corresponds to 3 cents in increased supply. Figure 1 in Section I provides a visual representation of this formula.

The model nests many traditional asset pricing models, including models with inelastic supply ($\zeta_S = 0$) and hence contribution multipliers of zero[2]. In contrast, I show that if supply is even slightly elastic then the contribution multipliers can be economically significant and exhibit substantial cross-sectional variation. This is important as non-trivial contribution multipliers could be the difference between an investor with environmental concerns holding coal stocks in their portfolio or not. Or, they could affect the investment decisions of a pension fund that wants to align itself with AI-driven growth but is concerned about accelerating disruptions to its beneficiaries livelihoods.

Having defined investor impact and demonstrated its dependence on contribution multipliers, I next derive the optimal strategy for an investor with impact preferences. Contrary to conventional wisdom, in my model even infinitesimal, uncoordinated investors (with impact preferences) internalize the investor impact of their portfolio choice. In essence, if supply is elastic, infinitesimal demand shifts generate infinitesimal price shifts which lead to infinitesimal impacts that can nevertheless be cost-effective from the perspective of an infinitesimal investor. This contrasts with Pástor et al. (2021) and related papers where inelastic supply means that investors do not internalize the

[2]For example, such assumptions have long been standard in the literature on asset pricing in endowment economies (Lucas, 1978).

effect of their choices on the sustainability-related good, or Oehmke and Opp (2020) where investor coordination is required[3].

The contribution multipliers drive a wedge between the optimal "impact tilt" and the tilt associated with holdings-based preferences, as illustrated in Figure 2 of Section I.B. The impact tilt can be understood in terms of "impact returns" that stand alongside expected financial returns in determining optimal portfolios. These impact returns, accounting for the contribution multipliers, can then be inserted as exogenous tilts into the models of papers like Pástor et al. (2021), Pedersen et al. (2021), and Zerbib (2022), generalizing these models to impact preferences. The magnitude of the impact returns is determined by an investor's "price of impact"—their marginal rate of substitution between the financial and the sustainability-related good.

The model also accounts for passive investors as in Betermier et al. (2022). The effect of passive investment is important as it means that active investors, whose changes in demand drive changes in supply, can have leveraged effects as passive investors follow along.

Following these general developments, I present a microfounded specification for the supply and demand elasticities, introducing a novel feature where firms' costs of investment depend on the amount of investment by other firms via a cost-sensitivity matrix. This is a way to account for competition and diminishing returns to scale at the industry level and is necessary to generate non-zero supply cross-elasticities. This microfounded case allows me to show that cross-sectional variation in demand and supply elasticities should be expected to generate a relatively positive covariance between expected financial and impact returns, in line with the findings of Cole et al. (2020) for the IFC's portfolio.

Next I discuss how an investor's "price of impact"—the marginal rate of substitution between the financial and sustainability-related goods—can be empirically determined. It can be inferred from observed impact tilts after accounting for firms' social productivities and associated contribution multipliers. Additionally, while the utility an investor derives from the sustainability-related good is subjective, if investors follow a coherent strategy across all allocations of capital, charitable cost-effectiveness estimates can be used to inform upper bounds on the price of impact. In other words, an investor should not be willing to pay more for impact via their investments than via charity.

After presenting the model, I use it as a framework to examine the contribution multipliers and

[3]See also the discussion in the appendix of Green and Roth (2021). See Gelman et al. (2012) for a related discussion of the impact of voting in a U.S. presidential election.

prices of impact that can be inferred from the relevant empirical literature. First, based on Betermier et al. (2022) and Koijen et al. (2022), I estimate an average, long-term contribution multiplier of 3% for large- and medium-sized U.S. stocks. This is economically significant yet consistent with the weak or null results from studies that have examined the real effects of investor demand shifts (Berg et al., 2023; Noh et al., 2023; Briere and Ramelli, 2022). Second, I present a table of contribution multipliers for firms across a range of supply and demand elasticities, highlighting the potential cross-sectional heterogeneity. In line with such heterogeneity, the real effects observed by Briere and Ramelli (2022) are greater for firms with lower credit ratings (higher associated demand elasticities). My results also highlight that variation in supply elasticities can significantly drive cross-sectional differences in contribution multipliers.

Third, I infer the prices of impact implied by the demand shifts observed for the various sustainability-related metrics examined in Koijen et al. (2022), Noh et al. (2023), and Betermier et al. (2022). The resulting prices of impact are all strikingly high, if not implausibly so. I also infer "bottom-up" prices of impact from reported social costs and charitable cost-effectiveness estimates for greenhouse gas emissions. These prices of impact are all lower than those inferred from the observed demand shifts. Together these results present a tension between the observed tilts and coherent impact preferences.

Several possibilities could resolve this tension. First, investors in the empirical sample may not be pricing impact according to my model, possibly due to emotional considerations (Heeb et al., 2022), Knightian uncertainty (Yasuda, 2023), or because they are generating impacts beyond the direct effects of their asset allocation. Second, some investors may view sustainability-related scores as financially informative and tilt towards higher-scoring firms for financial reasons. Third, the empirical results leveraged in this paper may not be valid for my application in some non-trivial way, requiring further refinement of the already impressive empirical methodologies employed in the relevant papers.

I also present two short extensions to my baseline model that demonstrate how contribution multipliers arise in other channels for investor impact. First, the possibility of investor influence on endogenous firm social productivities has been, for example, studied in Pástor et al. (2021). I show that this may be an important channel for impact via certain firms, but that in any case the potential for impact via this channel is moderated by an elasticity-dependent contribution multiplier. Second, in a simple model with search frictions where only one investor invests per firm, I show that a supply and demand-dependent contribution multiplier also emerges.

The overall contribution of this paper is to highlight how models that account for supply and demand heterogeneity can serve as a rich framework to address important questions beyond traditional asset pricing. This includes questions on topics that relate to sustainability-related outcomes, such as blended finance and universal ownership. I conclude the paper with a discussion of such potential applications and extensions.

This paper contributes to several literatures. It closely relates to the rapidly expanding body of work on models that allow for impact preferences (Oehmke and Opp, 2020; Baker et al., 2022b; Piatti et al., 2022; De Angelis et al., 2022; Broccardo et al., 2022; Hong et al., 2021; Landier and Lovo, 2020; Roth, 2021; Gupta et al., 2021; Chowdhry et al., 2019; Hart and Zingales, 2017). The paper's results, which link impact tilts to heterogeneous contribution multipliers, complement existing models in which the relevant contribution multipliers are fixed, dependent on investor size or coordination, or are not explicitly disentangled. My theoretical results align with Green and Roth (2021)'s conclusion that investors with impact preferences are likely to invest in ways that generate both greater impact and higher financial returns than investors with warm glow preferences. Moreover, the incorporation of impact preferences into an asset pricing model complements the vibrant literature on models with holdings-based preferences (Pástor et al., 2021; Berk and Van Binsbergen, 2021; Zerbib, 2022; Pedersen et al., 2021; Baker et al., 2022a; Fama and French, 2007; Avramov et al., 2021a; Avramov et al., 2021b; Goldstein et al., 2022; Luo and Balvers, 2017; Friedman and Heinle, 2016; Gollier and Pouget, 2022; Barnea et al., 2005; Heinkel et al., 2001).

The supply and demand system in this paper underscores the importance of the demand curve literature (Koijen et al., 2022; Koijen and Yogo, 2019; Haddad et al., 2021; Van der Beck, 2022; Gabaix and Koijen, 2021; Noh et al., 2023; Van der Beck, 2021; Merton, 1987), as well as the literature that incorporates an elastic supply side (Betermier et al., 2022; Bai and Zhang, 2022; Choi et al., 2022; Gonçalves et al., 2020; Bai et al., 2019; Ma, 2019; Belo et al., 2013). In principle, the supply and demand framing can be applied to any asset and, for example, the large real effects found in several studies of the impact of credit provision are consistent with high supply elasticities and low demand elasticities (Green and Vallee, 2022; Blouin and Macchiavello, 2019; Banerjee and Duflo, 2014).

My results also relate to the growing empirical literature demonstrating sustainability-related preferences driving investor decisions, fund flows and willingness-to-pay (Riedl and Smeets, 2017; Hartzmark and Sussman, 2019; Barber et al., 2021; Bauer et al., 2021; Heeb et al., 2022; Siemroth and Hornuf, 2021; Krueger et al., 2020). By relating the price of impact in my model to charitable opportunities and social costs, I not only complement other models with charitable opportunities (Graff Zivin and Small, 2005; Baron, 2007; Baron, 2009), but also contribute a generalized asset pricing model to the literature that examines altruistic investor strategies from first principles (Roth Tran, 2019; Baker et al., 2022b).

My findings support the arguments of Berk and Van Binsbergen (2021), Broccardo et al. (2022), and Davies and Van Wesep (2018) that divestment is unlikely to be a high-impact strategy, while adding the nuance that select cases of divestment may be coherent depending on an investor's price of impact. The profile of an ideal investment that emerges from my model—investments in highly profitable, socially productive firms with inelastic demand and elastic supply—matches the features of investments made by prominent investors with impact preferences (Cole et al., 2022; Kovner and Lerner, 2015). My model for impact returns offers an approach to analyze whether sustainability-related outcomes generated by impact investing compensate for lower financial returns, and contribution multipliers provide a more nuanced framework for assessing additionality than the binary evaluations common in prior literature (Carter et al., 2021; Cole et al., 2022).

In summary, this paper contributes to several strands of literature, including asset pricing models with holdings-based and impact preferences, supply and demand systems, and empirical research on sustainability-related preferences. The model presented offers a comprehensive framework for understanding the relationship between impact preferences, contribution multipliers, and asset prices, while providing valuable insights into the complexities and nuances of impact investing. It helps bridge the gap between theoretical models and empirical findings in the burgeoning field of sustainable finance, offering potential applications and extensions to areas such as blended finance and universal ownership.

This paper is organized as follows. Section I develops my baseline model for a generalized supply and demand system. Section II presents a microfounded case. Section III discusses the price of impact. Section IV presents empirical results. Section V develops extensions of the model that illustrate how contribution multipliers also arise in other cases. Section VI discusses the limitations of the model and potential further extensions. Section VII concludes.

I. Model

In this section, I introduce the model that serves as a framework for the rest of the paper[4].

I.A. Supply and Demand System

The model is a generalized supply and demand system that resembles the demand system of Van der Beck (2022) with the addition of an elastic supply side and a sustainability-related good. There are I investors indexed by $i \in \{1, \dots, I\}$, N firms indexed by $n \in \{1, ..., N\}$, and a single period between two times $t \in \{0, 1\}$.

At time 0, each firm has installed capital $K_0(n) \geq 0$ (after depreciation) and no other resources or liabilities. There is one share outstanding for each unit of installed capital and each investor initially holds $K_{i,0}(n)$ shares with $K_0(n) = \sum_{i=1}^{I} K_{i,0}(n)$. To fund investment and generate an economic profit for its initial shareholders, each firm increases its installed capital and shares outstanding to $K(n) = K_0(n) + \Delta K(n)$, sells all shares at price $P(n)$, and distributes the resulting cash flow $P(n)K(n) - I(n)$, proportionally to the initial shareholders. $I(n)$ is the cost of investment to install $\Delta K(n)$ of new capital. $K(n)$ may be interpreted as the firm's book value (in which case $P(n)$ is the market-to-book ratio of the firm).

The total production of the sustainability-related good is a function $G(\boldsymbol{K})$ of the installed capital of the firms. I define G so that it is positive in the direction that is considered "good" (so, for example, positive CO2 emissions produce negative G). In the region of the equilibrium I assume that $g(n) \equiv \frac{\partial G}{\partial K(n)}$ is an exogenous coefficient that defines the "social productivity" of the firm.

Each investor's wealth at time 0 consists of their initial holdings in each firm, $\boldsymbol{K_{i,0}}$, and cash C_i. In equilibrium each investor's wealth is equal to $W_{i,0}$ and the total initial wealth is $W_0 = \sum_{i=1}^{I} W_{i,0}$. Investors may invest in the shares of the N firms, as well as a risk-free asset with exogenous interest rate r_f. The risk-free asset has no direct effect on the sustainability-related good.

Each investor's demand curves $\boldsymbol{K}_i(\boldsymbol{P}, \boldsymbol{X})$ are an investor-specific function of the vector of current asset prices $\boldsymbol{P}$ and a collection of other model variables $\boldsymbol{X}$ (the choice of variables will depend on the microfoundations of the model). Aggregate demand is $\boldsymbol{K}_D = \sum_{i=1}^{I} \boldsymbol{K}_i$. Similarly, I

[4]Vectors and matrices are in bold and their elements are indicated in parentheses (e.g., $x(n, m)$ is the nth row and mth column of matrix $\boldsymbol{x}$). Negative row or column numbers indicate the removal of that row or column from the vector or matrix. $\boldsymbol{x}'$ indicates the transpose of $\boldsymbol{x}$. If $\boldsymbol{x}$ is a vector, then $diag(\boldsymbol{x})$ is the diagonal matrix with $\boldsymbol{x}$ on the diagonal, otherwise it is the vector of diagonal values of the matrix $\boldsymbol{x}$. I denote the identity matrix as $\boldsymbol{I}$ and a vector with the nth element equal to one and other elements equal to zero as $\boldsymbol{e}_n$.

stack each firm's supply curve into the vector-valued function $\boldsymbol{K}_S(\boldsymbol{P}, \boldsymbol{X})$. Market clearing implies that total asset demand equals total supply:

$$\sum_{i=1}^{I} \boldsymbol{K}_i(\boldsymbol{P}, \boldsymbol{X}) = \boldsymbol{K}_S(\boldsymbol{P}, \boldsymbol{X}) \tag{2}$$

Each investor's demand curves have an associated *absolute* price elasticity of demand matrix, $\tilde{\boldsymbol{\zeta}}_i$, with elements $\tilde{\zeta}_i(n,m) \equiv -\frac{\partial K_i(n)}{\partial P(m)}$. I refer to the diagonal elements as elasticities and the off-diagonal terms as cross-elasticities. The aggregate absolute price elasticity of demand matrix is $\tilde{\boldsymbol{\zeta}}_D = \sum_{i=1}^{I} \tilde{\boldsymbol{\zeta}}_i$. Similarly, but with a different sign, the absolute price elasticity of supply matrix $\tilde{\boldsymbol{\zeta}}_S$ has elements $\tilde{\zeta}_S(n,m) \equiv \frac{\partial K_S(n)}{\partial P(m)}$. As discussed in Van der Beck (2022) the demand elasticity matrix can be thought of as a function of trading costs, cash flow correlations and risk aversion, or investment constraints. The novel supply elasticity matrix in my model can be thought of as arising from capital adjustment costs and competition between firms[5]. Section II presents a microfounded model for both sets of elasticities. Further understanding these drivers is an important avenue for future research.

As in Van der Beck (2022) it will be useful to be able to refer to "price multiplier" matrices, $\tilde{\boldsymbol{\mathcal{M}}}_S = \tilde{\boldsymbol{\zeta}}_S^{-1}$ and $\tilde{\boldsymbol{\mathcal{M}}}_D = \tilde{\boldsymbol{\zeta}}_D^{-1}$, that are the inverse of the elasticity matrices. The diagonals of these matrices are the slopes of the supply and demand curves.

Investors may have heterogeneous beliefs and, as in Koijen et al. (2022), I assume that investors have full information about other investors' beliefs and agree to disagree. In particular, all agents are only required to agree about the market observables ($\boldsymbol{P}$, $\boldsymbol{K}$, and $\boldsymbol{K_i}$) and the aggregate supply and demand elasticity matrices. Investors may disagree about all other model parameters and I indicate this with a subscript i on such parameters as appropriate.

The supply and demand curves are endogenous as, similar to Haddad et al. (2021), investors are sophisticated and strategically update their demand curves to be optimal for their own expected utility given their beliefs about aggregate demand and supply. Other papers where investors explicitly optimize their demand schedule include Edmans et al. (2022) and Da (2022). The next proposition formally defines investor impact in terms of such strategic updates and shows how it can be approximated to first order.

[5] Rotemberg (2019) provides evidence for substantial variation in supply cross-elasticities between firms in different product categories

Proposition I.1. ***Investor Impact***. *The investor impact of investor* i*, defined as* $II_i \equiv G(\boldsymbol{K}|\boldsymbol{K_i} + \Delta\boldsymbol{K}_i) - G(\boldsymbol{K}|\boldsymbol{K_i})$*, is the change in the equilibrium expected value of the sustainability-related good* G *when the investor strategically shifts their demand curve from* $\boldsymbol{K}_i$ *to* $\boldsymbol{K}_i + \Delta\boldsymbol{K}_i$*. To first order, investor impact is given by*

$$II_i = \boldsymbol{g}_i'\boldsymbol{\mathcal{C}}\Delta\boldsymbol{K_i}, \tag{3}$$

where $\boldsymbol{\mathcal{C}} \equiv \tilde{\boldsymbol{\zeta}}_S(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}$ *is the "contribution multiplier matrix". Equivalently,* $\boldsymbol{\mathcal{C}} = (\tilde{\boldsymbol{\mathcal{M}}}_D + \tilde{\boldsymbol{\mathcal{M}}}_S)^{-1}\tilde{\boldsymbol{\mathcal{M}}}_D$ *based on basic matrix identities.*

See Appendix A.1 for the derivation. Figure 1 offers an intuitive visualization of investor impact for the case of a single firm. The key that makes this definition tractable when each investor is small is that such demand shifts, by $\Delta\boldsymbol{K}_i$, don't have first order effects on the aggregate demand elasticity and hence don't cause the demand curves of other investors to update.

The contribution multiplier matrix, $\boldsymbol{\mathcal{C}}$, determines how much an investor's demand shift $\Delta\boldsymbol{K}_i$ contributes to $\Delta\boldsymbol{K}$ (and ultimately to the sustainability-related good). Note that $\boldsymbol{\mathcal{C}}$ depends only on ratios between elasticities, not their levels. It is zero when either supply is assumed to be fully inelastic ($\tilde{\boldsymbol{\zeta}}_S = 0$) or demand is assumed to be fully elastic ($\mathrm{diag}(\tilde{\boldsymbol{\zeta}}_D) \to \infty$). These are explicit or implicit assumptions in many models. However, when neither of these assumptions are imposed investor demand shifts can generate investor impact. Note also that contribution multipliers are unrelated to the idea of "attribution" where one might seek to allocate a firm's overall impact across all investors —the contribution multipliers are simply about the marginal impact of each investor and in general the sum of investor impacts will be less than the overall production of the sustainability-related good.

Investor impact could be generated by any shift in demand whatever the underlying intentions. Demand shifts could be strategically chosen with the goal of generating a positive investor impact, or they could arise from changes to financial beliefs, transaction costs or other frictions and constraints. Thus, zooming out, it could be that a leading investor's work to lower the search and transaction costs associated with investing in highly socially productive firms does more to shift demand and generate investor impact than their own impact tilt—though such effects are out of scope for this baseline model.

It is important that investor impact is defined based on the change in the investor's entire demand function, $\Delta\boldsymbol{K}_i$, not just the change with respect to a single firm. As long as the cross-elasticities

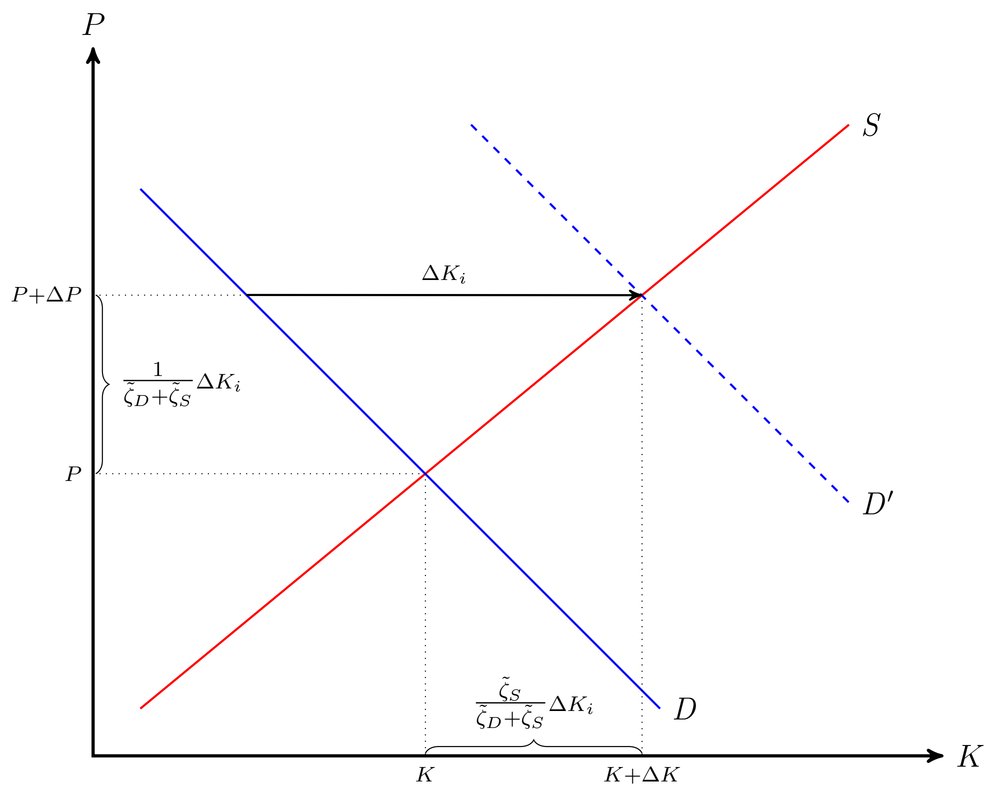

Figure 1. **Impact of a shift in an investor's demand curve**. This stylized diagram shows the effect of an individual investor shifting their demand for a single firm. All other investors' demand curves are assumed fixed so that the aggregate demand curve moves from D to D' based on the investor's chosen shift ΔK_i. The point at which the market clears moves along the firm's fixed supply curve S. The demand curve's slope is the inverse of the aggregate demand elasticity, $\tilde{\zeta}_D^{-1}$, and the supply curve's slope is the inverse of the aggregate supply elasticity, $\tilde{\zeta}_S^{-1}$. The equilibrium moves from (K, P) to $(K + \Delta K, P + \Delta P)$ with price shift $\Delta P = \frac{\Delta K_i}{\tilde{\zeta}_D+\tilde{\zeta}_S}$ and capital shift $\Delta K = \mathcal{C}\Delta K_i$, where $\mathcal{C} = \frac{\tilde{\zeta}_S}{\tilde{\zeta}_D+\tilde{\zeta}_S}$ is the "contribution multiplier" .

are non-zero a shift in investor demand for one firm will have spillover effects on other firms. For example, spillover effects could include unintentionally increasing demand for firms with negative social productivities (Van der Beck, 2022). This model can be used to calculate investor impact in a way that accounts for these non-trivial effects.

The following proposition shows how investor impact, and the contribution multiplier matrix $\boldsymbol{\mathcal{C}}$, can be broken down into intuitive components. This result align with how investor contribution is broken into components by practitioners (Impact Frontiers, 2023) and offers guidance for how these components might be assessed in practice.

Proposition I.2. *The investor impact of a demand shift $\Delta K_i(n)$, according to the beliefs of investor i, may be written as*

$$II_i(n) = g_i(n)\mathcal{C}_i^E(n)\mathcal{C}^I(n)\Delta K_i(n), \tag{4}$$

*with **investor contribution multiplier** $\mathcal{C}^I(n) \equiv \mathcal{C}(n,n) = \mathcal{C}_0^I(n)\mathcal{C}_{Cross}^I(n)$, where*

$$\mathcal{C}_0^I(n) = \frac{\frac{\tilde{\mathcal{M}}_D(n,n)}{\tilde{\mathcal{M}}_S(n,n)}}{1 + \frac{\tilde{\mathcal{M}}_D(n,n)}{\tilde{\mathcal{M}}_S(n,n)}}, \tag{5}$$

and

$$\mathcal{C}_{Cross}^I(n) = \frac{\left(1 - \frac{\tilde{\boldsymbol{\mathcal{M}}}_D(n,-n)}{\tilde{\mathcal{M}}_D(n,n)}\boldsymbol{\beta}_{T,n}\right)}{\left(1 - \frac{\tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)}{\tilde{\mathcal{M}}_T(n,n)}\boldsymbol{\beta}_{T,n}\right)}, \tag{6}$$

*and with **enterprise contribution multiplier***

$$\mathcal{C}_i^E(n) = \left(1 - \frac{\boldsymbol{g}_i(-n)'}{g_i(n)}\boldsymbol{\beta}_{T,n}\right), \tag{7}$$

where $\tilde{\boldsymbol{\mathcal{M}}}_T = \tilde{\boldsymbol{\mathcal{M}}}_D + \tilde{\boldsymbol{\mathcal{M}}}_S$ and $\boldsymbol{\beta}_{T,n} = (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1}\tilde{\boldsymbol{\mathcal{M}}}_T(-n,n)$.

See Appendix A for the derivations. These results highlight both the basic intuitions behind investor impact and the potential complexities. Note that while I have presented the results in terms of the price multiplier matrices, as they offer a particularly intuitive interpretation, similar formulas can be derived in terms of the elasticity matrices.

$\mathcal{C}^I(n)$ is broken out as the investor contribution multiplier because it can intuitively be understood as determining the effect of the investor's demand shift $\Delta K_i(n)$ on installed capital $K(n)$. It also doesn't depend on $\boldsymbol{g}$, unlike the enterprise contribution multiplier $\mathcal{C}^E(n)$. Furthermore, in Appendix A.3, I show that the enterprise contribution multiplier may be interpreted as the ratio between a firm's "enterprise impact", defined as the change in the sustainability-related good between an economy with and without the firm, and its "absolute impact" of $g(n)K(n)$.

The first part of the investor contribution multiplier, $\mathcal{C}_0^I(n)$, has a simple interpretation: investor contribution is greater for firms with more elastic supply and less elastic demand.

To develop intuition for the other terms, it is useful to imagine the matrix $\tilde{\boldsymbol{\mathcal{M}}}_T$ as the covariance matrix of a set of random variables, one for each firm as in Stevens (1998). Indeed, in the microfounded specification in Section II, $\tilde{\boldsymbol{\mathcal{M}}}_D$ is proportional to the covariance matrix of the firm's cash flows and $\tilde{\boldsymbol{\mathcal{M}}}_S$ is proportional to a matrix that controls the relationship between firms' investment costs (and could have many of the properties of covariance matrices).

Interpreting $\tilde{\boldsymbol{\mathcal{M}}}_T$ in this way, the demand shift $\Delta \boldsymbol{K}$ associated with $\Delta K_i(n)$ can be viewed as the optimal "portfolio" of demand shifts adopted by the market given "covariance" matrix $\tilde{\boldsymbol{\mathcal{M}}}_T$ and "excess expected return" vector $\tilde{\boldsymbol{\mathcal{M}}}_D(\cdot, n)\Delta K_i(n)$. The denominator in $\mathcal{C}_{Cross}^I(n)$ then reflects that the more the "portfolio covariance" can be reduced by offsetting positions in other assets, the more can be invested in the firm in question. The numerator in $\mathcal{C}_{Cross}^I(n)$ reflects the cost of implementing these "hedge" positions. This means that while more "replicable" firms will tend to yield greater contribution multipliers (given a smaller denominator), the overall effect depends on the numerator and remains ambiguous in general, without specifying particular parameter values.

Similarly, the enterprise contribution multiplier reflects the cost of these positions in terms of production of the sustainability-related good. A firm with a positive but mediocre $g_i(n)$ could have a negative enterprise contribution multiplier if it competes with firms with high values of g_i (i.e., if $\beta_{T,n}$ is large for such firms). Equally, if a firm has positive values of $\beta_{T,n}$ associated with firms with negative g, or negative values of $\beta_{T,n}$ associated with firms with positive g, then its enterprise contribution multiplier could be above one. Note that, defined in this way, the enterprise contribution multiplier depends, via $\beta_{T,n}$, as much on investment-side parameters as production-side parameters.

I.B. Optimal Demand Curves and Impact Returns

In the previous subsection, I showed that marginal investors can have investor impact and that this impact depends non-trivially on the contribution multiplier matrix. In this subsection, building on the possibility of investor impact, I derive the optimal policy of an investor with a taste for the sustainability-related good.

I assume that investor i's expected utility can be decomposed into:

$$U_i = U_{i,0}(\boldsymbol{K_i}) + U_i^H(\boldsymbol{K_i}) + U_i^G(\boldsymbol{K}) \tag{8}$$

with baseline financial utility $U_{i,0}(\boldsymbol{K_i}) = U_i^F(\boldsymbol{K_i}) - (1 + r_f)\boldsymbol{P'K_i}$, where U_i^F does not depend explicitly on $\boldsymbol{P}$ (for example, $U_i^F = \boldsymbol{a}'\boldsymbol{K}$ with $\boldsymbol{a}$ vector of firm profitabilities). $U_i^H(\boldsymbol{K_i}) \equiv \boldsymbol{h}_i'\boldsymbol{K}_i$, is a holdings-based preference term and $\boldsymbol{h}_i$ is a vector of exogenous holdings-based tastes that may be due to non-financial risk, subjective financial beliefs, and holdings-based sustainability preferences. The impact preference term is $U_i^G(\boldsymbol{K}) \equiv \gamma_i^g G$. Each utility term is expressed in units of the financial good. The investor's "price of impact', γ_i^g, defines the marginal rate of substitution between the sustainability-related good and the financial good.

In a competitive equilibrium no agent should be able to improve their utility by choosing a different feasible quantity for their demand (or supply), subject to the market clearing condition, with the supply and demand curves of the other agents held fixed. In my baseline case, assuming that each firm's characteristics are fixed so that the only endogenous variables are $\boldsymbol{K}_i$, $\boldsymbol{P}$, and $\boldsymbol{K}$, I derive the following proposition in Appendix B.

Proposition I.3. ***Optimal investor demand**. Suppose that $\boldsymbol{P}_{i,0}(\boldsymbol{K}_i)$ defines an investor's optimal demand curves when $\gamma_i^g = 0$ and $\boldsymbol{h}_i = 0$ (i.e., with only traditional financial preferences). Then, assuming investors are small enough that a first-order approximation is sufficient, for non-zero values of γ_i^g and $\boldsymbol{h}_i$ the optimal demand curves are given by*

$$\boldsymbol{P} = \boldsymbol{P}_{i,0}(\boldsymbol{K}_i) + \frac{1}{1 + r_f}\Big(\boldsymbol{h}_i + \gamma_i^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}_i\Big). \tag{9}$$

This result shows that both holdings-based and impact preferences lead to shifts in the investor's demand curves and that impact preferences are equivalent to holdings-based preferences with $\boldsymbol{h}_i = \gamma_i^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}_i$. It also means that impact preferences can generate an economically significant shift in investor demand given sufficiently positive values for the price of impact (γ_i^g), the social productivities ($\boldsymbol{g}_i$), and the contribution multiplier matrix. In the empirical section of this paper I examine the evidence for whether or not this is the case for popular sustainability-related metrics in the U.S. stock market. Figure 2 provides a visual representation of the distinction between the demand tilt generated by impact preferences and that generated by holdings-based preferences.

In general, many of the elements in the contribution multiplier matrix can be expected to have values substantially lower than 1, given the standard assumption that supply elasticities are much lower than demand elasticities. The matrix arises under a first order approximation and is primarily valid for investors that are small compared to the relevant market. In contrast, for a representative investor the contribution multiplier matrix will be the identity matrix. Thus, if all investors (or a substantial majority) were able to agree on their nonpecuniary preferences and coordinate their actions, then the contribution multipliers would not be a concern.

Such coordinated investors would likely allocate considerably more capital to firms with high social productivity than uncoordinated investors. If there was broad agreement on nonpecuniary values then this would suggest a "price of anarchy" to investor's lack of coordination. However, both common sense and the heterogeneity observed in Koijen et al. (2022) suggest that investors are far from fully coordinated in this regard. As a result, if there is a price being paid, it could well be being paid by investors with strong nonpecuniary preferences who assume it is possible to coordinate with the market at large.

To understand what investors are willing to pay to optimize for their preferences, consider that Proposition I.3 suggests that the investor acts as if they expect pseudo-returns

$$\boldsymbol{IR}_i = \frac{\gamma_i^g}{(1+r_f)} \operatorname{diag}(\boldsymbol{P})^{-1} \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i, \tag{10}$$

which I refer to as "**impact returns**", in addition to financial returns. The impact return for firm n is then

$$IR_i(n) \equiv \frac{\gamma_i^g g(n) \mathcal{C}^E(n) \mathcal{C}^I(n)}{(1+r_f) P(n)}, \tag{11}$$

and the associated "impact Sharpe ratio" is $\lambda_i^g(n) \equiv IR_i(n)/\sigma(n)$, where $\sigma(n)$ is the volatility of firm n's stock returns.

These definitions of impact returns and impact Sharpe ratios are key to my empirical analysis. If impact returns can be observed, along with estimates of the contribution multipliers, then this

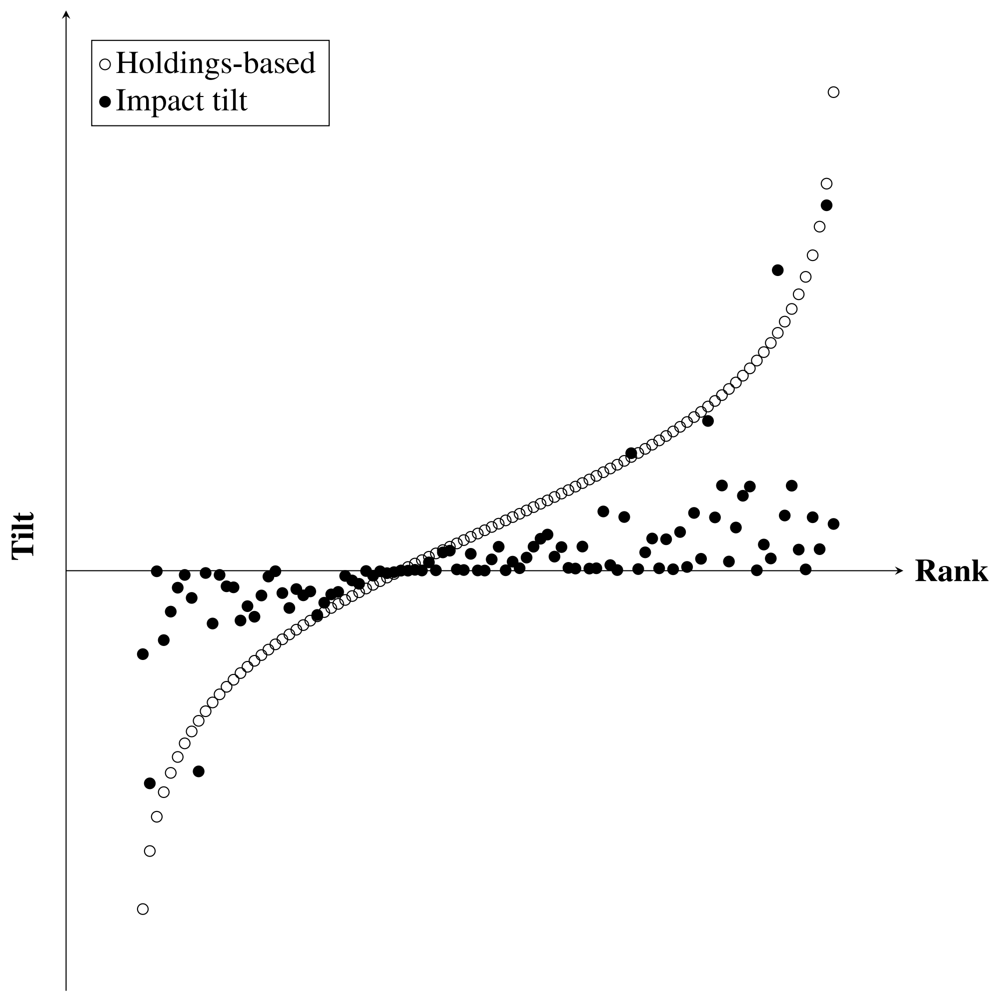

Figure 2. **Impact tilt compared to corresponding holdings-based tilt**. This figure presents a stylized chart of the optimal demand shift or "impact tilt" associated with impact preferences ($Tilt \propto \boldsymbol{\mathcal{C}}'\boldsymbol{g}$) and the corresponding tilt for holdings-based preferences ($Tilt \propto \boldsymbol{g}$). Assets are ranked from left to right in order of increasing social productivity, g. The impact tilt, which accounts for the contribution multiplier matrix $\boldsymbol{\mathcal{C}}$, generally has a smaller magnitude than the holdings-based tilt. However, it can vary considerably depending on the relevant supply and demand elasticities. In some cases, its magnitude can exceed that of the holdings-based tilt. This illustrates how impact preferences are fundamentally different from holdings-based preferences.

can inform an estimate of investors' prices of impact, γ_i^g. The assessed prices of impact can inform discussion of whether investors are following holdings-based or impact preferences.

The shift in the demand curve from turning on the investor's impact returns is their "impact tilt" portfolio,

$$\Delta \boldsymbol{K}_i^{II} = \gamma_i^g \tilde{\boldsymbol{\zeta}}_i \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i, \tag{12}$$

which generates an investor impact of $II_i^* = \gamma_i^g \boldsymbol{g}_i' \boldsymbol{\mathcal{C}} \tilde{\boldsymbol{\zeta}}_i \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i$. What should the impact-tilt portfolio be expected to look like? Heavy-tailed distributions of $\boldsymbol{g}_i$ are a stylized fact in the cost-benefit literature (see, for example, Gillingham and Stock (2018), Angrist et al. (2020), and Laxminarayan et al. (2006)). Additionally, as I show in Section IV.A, the values in $\boldsymbol{\mathcal{C}}$ are likely to vary over orders of magnitude in the cross section. These stylized facts suggest that the distribution of impact returns will have a highly skewed distribution. This implies that investors' impact tilts are likely to be concentrated on a few firms with exceptional impact returns.

Furthermore, investors subject to fixed transaction costs can be expected to prioritize investments in proportion to the level of their utility contribution: $\Delta U_i \propto \gamma_i^g II_i^* = (\gamma_i^g)^2 \boldsymbol{g}_i' \boldsymbol{\mathcal{C}} \boldsymbol{\zeta}_i \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i$. Because the parameters in this expression are all squared, its distribution across firms can be expected to be more skewed than the distribution of impact returns themselves. Hence, impact investors with high, fixed transaction costs are likely to have even more selective impact tilts.

Note that these points do not imply that investors with impact preferences will purely hold "impact investments". This could be the case for sufficiently high prices of impact. However, for more moderate cases, what seems more likely is that the handful of firms with economically significant impact returns will only be relatively over (or under) weight in an investor's portfolio. In Section III, I discuss how the price of impact might be determined.

I.C. Accounting for Passive Investors

As highlighted by Betermier et al. (2022) in an extension of their model, if shifts in demand by active investors cause supply to change, and passive investors react to these changes (i.e., to continue holding the market portfolio), then it is possible that the effects of active investors may be leveraged.

To account for this, I assume that the set of investors may be partitioned into a subset of active investors, A, and a subset of passive investors, P. Each passive investor seeks to hold

$\boldsymbol{K}_P = \boldsymbol{\kappa}_P \boldsymbol{K}_A$, with $\boldsymbol{\kappa}_P$ a diagonal matrix that summarizes the multiple of the active portfolio they choose to hold for each stock. These multiples will depend on a mix of, for example, the relative wealth and risk aversion of the passive investors as in Betermier et al. (2022), the demand structures explored in Gabaix and Koijen (2021), or the benchmarking intensity of Pavlova and Sikorskaya (2023).

These assumptions imply that $\boldsymbol{K}_D = (\boldsymbol{I} + \boldsymbol{\kappa}_P)\boldsymbol{K}_A$ and thus that $\tilde{\boldsymbol{\zeta}}_D = (\boldsymbol{I} + \boldsymbol{\kappa}_P)\tilde{\boldsymbol{\zeta}}_A$, where $\tilde{\boldsymbol{\zeta}}_A$ is the aggregate demand elasticity matrix of the active investors. This means that for an active investor i, the contribution multiplier matrix is given by

$$\boldsymbol{\mathcal{C}} \equiv \tilde{\boldsymbol{\zeta}}_S(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}(\boldsymbol{I} + \boldsymbol{\kappa}_P). \tag{13}$$

More generally, if an active investor i has a population of investors that follow their lead, so that a change in their demand of $\Delta \boldsymbol{K}_i$ generates total raw change in demand $(\boldsymbol{I} + \boldsymbol{\kappa}_i)\Delta \boldsymbol{K}_i$, then their contribution multiplier matrix will be $\boldsymbol{\mathcal{C}} \equiv \tilde{\boldsymbol{\zeta}}_S(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}(\boldsymbol{I} + \boldsymbol{\kappa}_i)$. In the language of impact investment practitioners, $\boldsymbol{\kappa}_i$ may be understood as the degree to which the investor is "catalytic".

II. Microfounded Case

To illustrate the implications of the model in an intuitive setting, in this section I present a particular microfounded specification. Each investor has standard mean-variance utility plus a taste for the sustainability-related good,

$$U_i = E[W_{i,1}] - \frac{\gamma_i}{2W_{i,0}} Var[W_{i,1}] + \gamma_i^g G, \tag{14}$$

where $W_{i,1}$ is the investor's wealth at time 1 and γ_i defines their level of financial risk aversion (or "price of risk").

At time 1, each firm generates free cash flow $CF(n) = z_{CF}(n)K(n)$. $\boldsymbol{z}_{CF}$ is a random variable with exogenous mean $\boldsymbol{a}_{CF}$ and covariance $\boldsymbol{\Sigma}_{CF}$. To keep the model self-contained within the single period, I assume that the cash flows capture not just profits from production during the period but also the ongoing market values of the installed capital.

As in Section I.C, I assume a split between active and passive investors with passive investors holding portfolio $\boldsymbol{K}_P = \boldsymbol{\kappa}_P \boldsymbol{K}_A$. Furthermore, as in Merton (1987) and Koijen et al. (2022), each active investor allocates $W_{i,0}$ at time 0 across assets in its opportunity set $\mathcal{N}_i \subseteq \{1, ..., N\}$ and the risk-free asset. The opportunity set may be a subset of the entire investment universe for a variety of underlying reasons, including lack of information, limitations on short sales, taxes, transaction costs, and other frictions, as well as divestment commitments. I denote the number of assets in the investor's opportunity set as $|\mathcal{N}_i|$. To help define the aggregate demand curves, let $\boldsymbol{I}_i$ be an $|\mathcal{N}_i| \times N$ matrix where for each $n \in \mathcal{N}_i$ the corresponding row in $\boldsymbol{I}_i$ is $\boldsymbol{e}_n'$.

Each firm's technology exhibits decreasing returns to scale and adjustment costs, both in regards to the firm itself and its competitors. As for example in Zhang (2017), I capture this as an investment cost that increases the amount of investment necessary to install $\Delta K(n)$ of new capital. In addition to a basic cost of $\Delta K(n)$ to install the new capital, I assume that the investment costs are

$$\Phi(n) = \frac{1}{2}\Sigma_C(n,n)\Delta K(n)^2 + \sum_{m \neq n} \Sigma_C(n,m)\Delta K(n)\Delta K(m), \tag{15}$$

where $\boldsymbol{\Sigma_C}$ is a matrix of cost sensitivities[6]. These sensitivities could arise due to competition for resources within an industry or externalities between firms in different industries. As in Zhang (2017), it could be that $\Sigma_C(n,n) \propto K_0(n)^{-1}$ so firms have economies of scale with respect to already installed capital. For simplicity, I assume in this paper that $\boldsymbol{\Sigma_C}$ is symmetric and invertible. This assumption makes sense, for example, if investment costs depend on the total new capital set to be installed in an industry, independent of how much each individual firm is installing. In general, though, there could be asymmetric effects.

I assume that each firm is managed at time $t = 0$ on behalf of existing shareholders who consume the economic profit—the difference between the firm's market value and its investment costs. The utility of firm n is thus:

$$U_F(n) = P(n)K(n) - \Delta K(n) - \Phi(n). \tag{16}$$

As in my baseline model, I assume firms' social productivities are exogenously specified, because I am uncertain about how to microfound the relevant parameters of the model for endogenous social productivities in Section V, in particular the cost of change.

For the above specification I derive the following proposition in Appendix C.

[6]This can be viewed as simplifying models that have both decreasing returns to scale and adjustment costs into a model with constant returns to scale and only adjustment costs.

Proposition II.1. ***Optimal microfounded supply and demand curves***. *To first order, the optimal demand curves are*

$$\boldsymbol{P} = \frac{(\boldsymbol{a}_i + \gamma_i^g \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i)}{1 + r_f} - \boldsymbol{\zeta}_i^{-1} \boldsymbol{K}_i, \tag{17}$$

and the optimal supply curves are

$$\boldsymbol{P} = \boldsymbol{1} + \boldsymbol{\zeta}_S^{-1}(\boldsymbol{K} - \boldsymbol{K}_0), \tag{18}$$

with supply and demand elasticity matrices

$$\boldsymbol{\zeta}_i = (1 + r_f)\frac{W_{i,0}}{\gamma_i} \boldsymbol{\Sigma}_{CF,i}^{-1}, \tag{19}$$

$$\boldsymbol{\zeta}_D = (\boldsymbol{I} + \boldsymbol{\kappa}_P)\left(\sum_{i \in A}^{I} \boldsymbol{I}_i' \boldsymbol{\zeta}_i \boldsymbol{I}_i\right), \tag{20}$$

$$\boldsymbol{\zeta}_S = \boldsymbol{\Sigma}_C^{-1}. \tag{21}$$

The supply elasticity matrix is determined by the firm cost-sensitivity matrix and firms with higher adjustment costs will be less elastic and have lower contribution multipliers. Firms with more volatile cash flows or that are in fewer investors' opportunity sets will tend to have lower demand elasticities and higher contribution multipliers.

Suppose the investor tilts their portfolio not necessarily by the optimal impact tilt but by a vector $\boldsymbol{\mu}_i^g$. The relevant part of their utility function is

$$U_i^{(g)} = \frac{W_{i,0}}{\gamma_i}\left(\gamma_i^g \boldsymbol{g}_i' \boldsymbol{\mathcal{C}} \boldsymbol{\Sigma}_{CF,i}^{-1} \boldsymbol{\mu}_i^g - \frac{1}{2}(1 + r_f)(\boldsymbol{\mu}_i^g)' \boldsymbol{\Sigma}_{CF,i}^{-1} \boldsymbol{\mu}_i^g\right). \tag{22}$$

This utility term is indeed maximized with the optimal demand curves and is equal to $U_i^{(g)} = \frac{1}{2}\frac{W_{i,0}}{(1+r_f)\gamma_i}(\gamma_i^g)^2 \boldsymbol{g}_i' \boldsymbol{\mathcal{C}} \boldsymbol{\Sigma}_{CF,i}^{-1} \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i$. Note that, heuristically, $U_i^{(g)}$ will be negative if the investor's tilt $\boldsymbol{\mu}_i^g$ is more than a factor of two greater than the optimal tilt (or of the opposite sign). Thus, it may be much worse to overestimate rather than underestimate the appropriate demand shift.

II.A. Relationship Between Impact and Financial Returns

The relationship between expected financial returns and "impact" is a topic of ongoing debate and mixed results (Starks, 2023). This is important because investors with different beliefs about this relationship may make different strategic decisions. An investor who expects a positive correlation between impact and financial returns is likely to search for and hold a portfolio of firms with both significantly positive a_i and g_i. Whereas an investor that believes in a negative correlation will be likely to hold a mix of "non-sustainable" firms for their superior financial returns and concessionary impact investments for their superior impact returns. These different types of investors are likely to devote their search to different parts of the overall opportunity set.

A useful framing with which to examine this relationship is in terms of an "Security Impact Line", as in the following proposition. This is the "impact" analog of the Security Market Line.

Definition II.1. *Investor i's "Security Impact Line" is the line defined by,*

$$EFR_i(n) + IR_i(n) = 0, \tag{23}$$

where the (risk-adjusted) "excess financial return"

$$EFR_i(n) \equiv \frac{(\boldsymbol{a}_i(n) - (1+r_f)\boldsymbol{\zeta}_i^{-1}(n,-n)\boldsymbol{K}_i(-n) - (1+r_f))}{P(n)}, \tag{24}$$

is equal to the expected return on the stock, minus the risk-free rate, minus a risk adjustment for a stock's systematic risk, and where the impact returns, $IR(n)$, are as in equation (11).

The security impact line differentiates between investment opportunities where the investor would desire a positive position (i.e., $EFR_i(n) + IR_i(n) > 0$ after accounting for impact returns), and opportunities where holding a negative position would be optimal ($EFR_i(n) + IR_i(n) < 0$). See Appendix D for the derivation. The slope of the security impact line depends on the price of impact; see Section III for further discussion of this parameter and a visualization of potential security impact lines.

In general, the relationship between the impact and excess financial returns is ambiguous and will depend on the relationship between the a_i and the g_i (and this relationship is out of scope for the present paper). However, all else equal, these returns may be correlated due to their shared dependence on prices and hence the supply and demand elasticities. In Appendix C.3, I show that equilibrium prices are (for simplicity, ignoring the effect of the initial firm sizes):

$$\boldsymbol{P} \approx \frac{(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}\tilde{\boldsymbol{\zeta}}_D}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}), \tag{25}$$

where $\boldsymbol{a}$ and $\boldsymbol{g}$ are weighted average vectors across all investors. The equilibrium firm sizes will be $\boldsymbol{K} = (\tilde{\boldsymbol{\zeta}}_D^{-1} + \tilde{\boldsymbol{\zeta}}_S^{-1})^{-1}\left(\frac{1}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) - \mathbf{1} - \tilde{\boldsymbol{\zeta}}_S^{-1}\boldsymbol{K}_0\right) = (\tilde{\boldsymbol{\zeta}}_D^{-1} + \tilde{\boldsymbol{\zeta}}_S^{-1})^{-1}\boldsymbol{\mu}$ with $\boldsymbol{\mu} = \frac{1}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) - \mathbf{1}$. Thus, for the standard case of relatively high demand elasticity, changes in prices will be a bigger driver of variations in returns than changes in $\boldsymbol{K}_i$ and the risk adjustment.

As long as a firm's profitability, a, is the main driver of its price (and not its social productivity g), an increase in the demand elasticity associated with a firm can thus be expected to both decrease the relevant contribution multiplier and increase prices, decreasing both financial and impact returns. In contrast, an increase in the supply elasticity associated with a firm can be expected to decrease prices and increase the relevant contribution multiplier, increasing both financial and impact returns. Overall, this indicates that the supply and demand system generates a positive relationship between financial and impact returns. This aligns with Green and Roth (2021)'s theoretical result that investors with impact preferences are likely to invest in ways that generate better financial and impact results than investors with warm glow preference.

Potentially offsetting the positive relationship that arises from the supply and demand system are variations in and relationships among $\boldsymbol{a}, \boldsymbol{g}, \boldsymbol{a}_i$ and $\boldsymbol{g}_i$. For example, increases in either a or g, with the other parameter held fixed, will tend to increase the return associated with the changing parameter and decrease the return associated with the other parameter. For fixed a_i and g_i, increasing a or g (the expectations of other investors) will increase the price and decrease expected returns for the investor. Thus, in general, the cross-sectional relationship will be ambiguous.

The connection between expected financial and impact returns, along with the expression for the impact tilt in equation (12), implies that the archetype of an ideal impact investment involves taking a large, positive position in a highly profitable, socially productive firm that is associated with highly elastic supply and inelastic demand. Cole et al. (2020) offers an empirical example of such a relationship in the portfolio of the International Finance Corporation (IFC). In other words, if investors can achieve both financial and impact objectives simultaneously, there is no reason not to pursue this strategy. Of course, in reality, highly profitable firms may attract significant levels of elastic demand, rendering the ideal difficult to attain. Nevertheless, among firms with the same level of profitability and social productivity, investors should prefer those associated with less elastic demand.

III. The Price of Impact

A key part of my model is that each investor's "price of impact", γ_i^g, determines their taste for the sustainability-related good and hence their optimal demand shift. An investor's price of impact can be determined in two ways. One approach is based on revealed preferences, where, given assessments of social productivities and contribution multipliers, the price of impact can be inferred from the impact returns implied by observed portfolio tilts. Alternatively, this parameter can be inferred by examining investors' choices regarding high-impact concessionary opportunities, particularly charitable opportunities. This section develops the latter approach, which, along with the former, will be applied in Section IV. For other models considering the option of charitable donations alongside investments, see Graff Zivin and Small (2005), Baron (2007), and Baron (2009).

An investor's price of impact is not a normal "price" in that it is a subjective parameter and not based on any market for buying and selling "impact" (which doesn't exist in the model and may not be possible to create in practice). In this paper I present the price of impact as exogenous. However, in general one can imagine it being set endogenously by sophisticated investors based on a combination of their beliefs about current and future opportunities as well as other preferences.

The price of impact, as the investor's marginal rate of substitution between the financial good and the sustainability-related good, will in practice be determined more by the rate of substitution that can actually be achieved, rather than subjective choices of the investor. That is, it will be determined by an investor's opportunity costs given all the options for this substitution that they have available. For example, if opportunities at future times are forecast to offer an abnormally favorable rate of substitution (e.g., due to higher future social productivities), then setting a low value for γ_i^g in the current period, reducing their current impact tilt and increasing their expected wealth, could maximize their expected utility over time. Only if the investor's subjective beliefs mean they aren't willing to utilize this forecast rate of substitution would an even lower price of impact be appropriate.

If the investor is a private individual, the set of available charitable opportunities is vast. For an institutional investor, fiduciary duties and operating procedures may limit the available opportunities, though sponsoring research reports or shareholder engagement initiatives may be an option. For some fiduciaries (e.g., asset managers, pension funds) it may be appropriate, for this exercise, to consider the charitable opportunities available to their clients[7].

In any case, suppose that CE_i^* is investor i's assessment of the cost-effectiveness (cost per unit of the sustainability-related good) of a benchmark charitable opportunity. Development finance institutions have indeed proposed using such benchmarks (Carter, 2021). In equilibrium, the investor should not be able to gain utility by donating more to this charity. Assuming their marginal investment is the risk-free asset, this implies that

$$\gamma_i^g \leq (1 + r_f) CE_i^*, \tag{26}$$

is an upper bound on the price of impact. An investor who uses a higher price of impact than this will be paying more for impact than the available opportunities require.

The price of impact need not be at this upper bound. That is, $\gamma_i^g < (1 + r_f) CE_i^*$ may hold. For example, an investor may choose to not have any impact tilt (or a weaker one) in the present time period. This could be due to a lack of impact preferences. Or, it could be because they forecast higher impact opportunities in the future and wish to save to fund these future opportunities.

More generally, for investors who do not fund explicitly charitable opportunities, one might consider a candidate "best available concessionary opportunity" (BACO) with (risk-adjusted) excess financial return EFR_i^*, and "investor impact per dollar"

$$IID_i^* = \frac{\mathcal{C}^* g^*}{(1 + r_f) P_*}, \tag{27}$$

given social productivity g^*, and contribution multiplier $\mathcal{C}^*$. The assumption that the investor can invest in this opportunity but chooses not to, on the margin, implies that the opportunity is on the wrong side of the security impact line. That is, $EFR_i^* + \gamma_i^g IID_i^* \leq 0$. This implies an upper bound for the price of impact of

$$\gamma_i^g \leq -\frac{EFR_i^*}{IID_i^*}. \tag{28}$$

Note that if this upper bound is negative, it would suggest that either the candidate BACO is not

[7]For example, the asset manager Fidelity Charitable made over $9 billion in donor-recommended grants in 2020 to 170,000 organization via donor-advised funds (https://www.fidelitycharitable.org/ insights/2021-giving-report.html).

appropriate as a benchmark (i.e., because it is an anomaly and will disappear with significant investment) or that the investor's marginal asset is not the risk-free asset (because the supply of high expected return assets exceeds the investor's demand). Figure 3 presents a stylized illustration of the relationship between an investor's "security impact line", their price of impact and different BACO candidates.

The price of impact γ_i^g in my model is different from and complementary to the use of social costs in the large literature on cost-benefit analysis. Investors may use social costs, for example, to attempt to compare different sustainability-related goods based on careful consideration of societal preferences. The price of impact, instead, is what the investor uses to optimize the strength of their impact tilt based on their opportunity set.

Another perspective on this relationship is that estimates of social costs are just another potential upper bound on γ_i^g. Indeed, in many cases it would be hard to justify spending more to avert an externality than society (i.e., the beneficiary) is itself willing to pay—though the investor could believe that social costs are underestimated. Whether or not the relevant social cost upper bound is lower than the charitable bound is an empirical question. By and large, however, I would expect that an unconstrained investor should be able to find opportunities that generate impact for less than the social cost.

Note that the differences between social costs and alternative bounds can be large. For example, the value of a statistical life in the U.S. is assessed to be on the order of magnitude of $10 million[8]. Whereas, the charity evaluator GiveWell consistently estimates that the charities they recommend save lives (in the developing world) at a rate of less than ten thousand dollars per life saved[9].

In some instances, the social cost might be more relevant for forecasting firm profitability than for determining the price of impact. Consider a situation where the government deems the social costs of an externality to be high and consequently implements policies to incentivize products addressing the externality (e.g., electric vehicle subsidies). If an investor believes the government has overestimated the social costs or provided excessively generous subsidies, they may not want to set their private γ_i^g based on these policies. However, they might see a financial opportunity in allocating capital to a commercial response.

[8]https://www.transportation.gov/office-policy/transportation-policy/revised-departmental-guidance-on-valuation-of-a-statistical-life-in-economic-analysis. Accessed: 2022-10-31.

[9]See, for example: https://www.givewell.org/cost-to-save-a-life. Accessed: 2022-10-31.

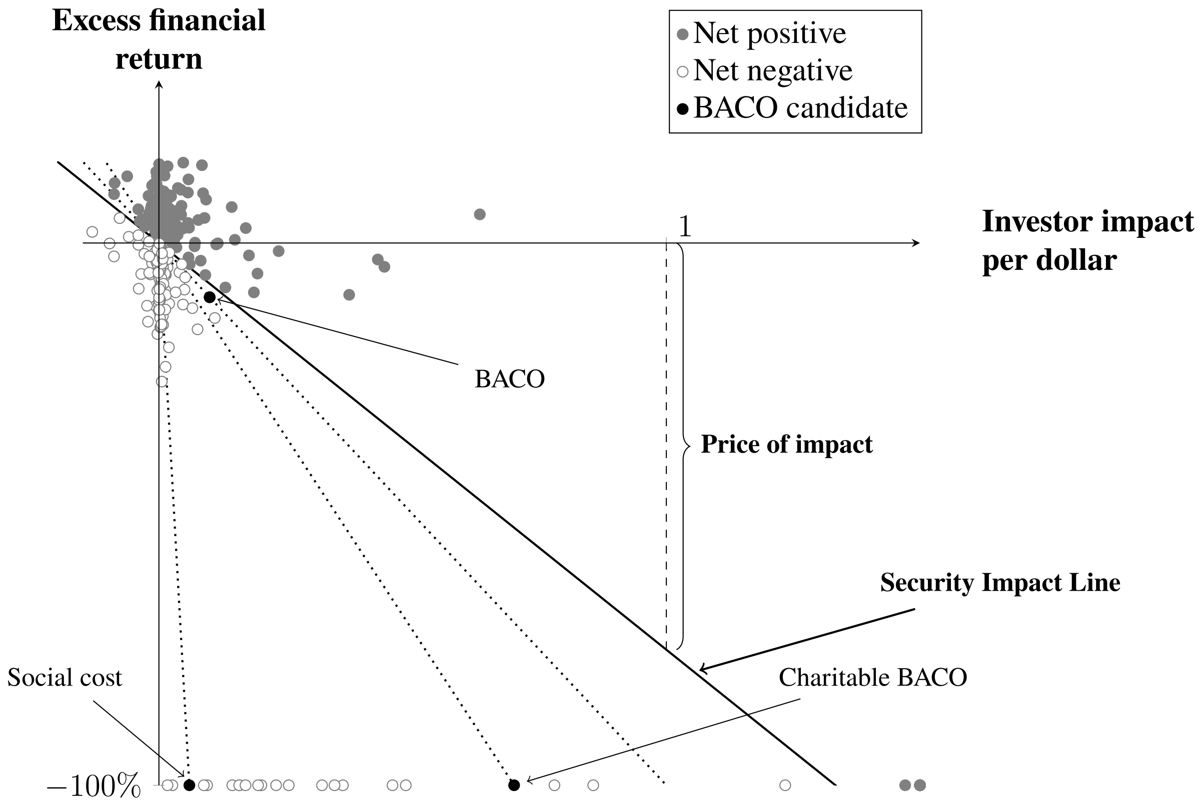

Figure 3. **Security Impact Line and the Price of Impact**. This figure presents a stylized representation of the "Security Impact Line", which distinguishes between net positive (solid gray circles) and net negative (empty circles) investment opportunities with respect to an investor's utility. The horizontal axis represents the "Investor impact per dollar", while the vertical axis denotes the risk-adjusted "Excess financial return", as defined in the main text. The "price of impact" corresponds to the slope of the Security Impact Line and can also be understood as the excess financial return associated with an investor impact per dollar value of 1 along this line. Any candidate "best available concessionary option", that is capable of absorbing significant additional investment, can set a lower bound on the Security Impact Line and an upper bound on the price of impact. This concept is illustrated for a hypothetical charitable opportunity with cost-effectiveness equal to the value of a unit of impact in a standard cost-benefit analysis ("Social cost"), the most cost-effective charitable candidate ("Charitable BACO"), and a non-charitable candidate ("BACO"). The Security Impact Line is displayed above these bounds (on the right of the vertical axis) to illustrate that the investor's price of impact is bounded by these BACO candidates, but not necessarily defined by them. For example, it may additionally depend on to their forecast of the cost-effectiveness of generating investor impact in the future (if that suggests a lower price of impact).

IV. Empirical Applications

In this section, I combine results from Betermier et al. (2022), Koijen et al. (2022) and Merton (1987) to offer initial results on how investors are pricing impact and how investor contribution multipliers may vary across firms. Comprehensive empirical study, especially the estimation of heterogeneous supply elasticities and cross-elasticities, presents many challenges and I leave such investigations as an important avenue for future research.

IV.A. Investor Contribution Multipliers

Homogeneous estimate. Betermier et al. (2022) estimate the supply and demand curves that apply to a panel of 1,226 large- and medium-sized U.S. firms over the period 1999-2019. They estimate the following econometric specification using three-stage least squares[10]:

$$\lambda_{n,t} = \boldsymbol{\beta}'_D \boldsymbol{x}_{D,n,t} + \Delta_D \rho_{M,n,t} + u_{D,n,t}, \tag{29}$$

$$\lambda_{n,t} = \boldsymbol{\beta}'_S \boldsymbol{x}_{S,n,t} + \Delta_S \rho^{int}_{M,n,t} + u_{S,n,t}, \tag{30}$$

where $\lambda_{n,t}$ is the Sharpe ratio, $\boldsymbol{x}_{D,n,t}$ and $\boldsymbol{x}_{S,n,t}$ are vectors of firm characteristics that are expected to drive supply and demand, $\rho_{M,n,t}$ is the correlation with the market portfolio, and $\rho^{int}_{M,n,t}$ is the portion of $\rho_{M,n,t}$ estimated to come from a stock's correlation with itself (used as a measure of firm size). Homogeneous slopes Δ_D and Δ_S, and linear coefficients $\boldsymbol{\beta}_D$ and $\boldsymbol{\beta}_S$, are imposed across firms. The estimation takes place on five-year rolling windows and thus is appropriate for estimating long-term supply and demand elasticities. The Sharpe ratio is estimated on the period after t, while the correlations are estimated on the prior period.

This specification implies homogeneous investor contribution multiplier $\mathcal{C}^I = \Delta_D/(\Delta_D - \Delta_S)$. As reported in Table I, the slope estimates by themselves imply an investor contribution multiplier of 0.9%. However, the actual investor contribution multiplier will depend on the wealth ratio between active and passive investors as in equation (13). Based on the analysis of active shares over time in Koijen et al. (2022), I use 35% as the active share for my analysis as this roughly corresponds to the active share during the middle of the sample periods of both Betermier et al. (2022) and Koijen et al. (2022). Combined with this active share estimate, the slope estimates reported in Table I

[10]Note that they frame their model in terms of the supply and demand for capital, not assets, so I have swapped the labeling of their equations to match the meaning of supply and demand in the present paper.

imply an investor contribution multiplier of 2.6%. Note that this estimate ignores the more complex terms in equation (6), though I expect that accounting for these terms would more likely than not decrease the estimated contribution multiplier (increasing the tension between impact preferences and observed tilts that I find later in this section).

There is substantial uncertainty in the estimate of the supply slope. In theory, this leaves open the possibility of much lower supply slopes and much higher investor contribution multipliers. However, estimates for the microfounded model of Section II are in line with the ratio between the supply and demand slopes reported here. For example, models with decreasing returns to scale and adjustment costs, all can be viewed as implying supply slopes that are approximately proportional to $1/K_0$. Whereas the demand slope for a firm that is in most investors opportunity sets will be proportional to $1/W_0$. For Betermier et al. (2022), K_0 is on the order of \$1-10 billion, whereas the relevant market wealth is greater than \$10 trillion. Thus, as a starting point one can expect a supply to demand slope ratio of at least 1000 to 1. The actual supply to demand slope ratio will depend on the exact parameters of aggregate risk aversion, cash flow volatility, adjustment costs and diminishing returns to scale, but based on existing empirical estimates (including from Bai and Zhang (2022), Choi et al. (2022), Bai et al. (2019), and Hennessy and Whited (2007)), it seems unlikely that these parameters will decrease this ratio by more than a factor of 10 for the average firm. More likely is that variation in the parameters of supply and demand across firms will drive variation in the investor contribution multiplier around this average ratio, as I next discuss.

Potential variation in investor contribution. In Table II, I extend an analysis of Merton (1987) to show how investor contribution might vary for firms with different supply and demand elasticities. Merton (1987) sorts the sample of 1,387 U.S. firms in his 1985 dataset into 10 groups with the same number of firms in each group. He then uses variation in market value per registered shareholder to form rough estimates of the relative size of the investor base of each group. That is, the ratio of the percentage of investors with a firm in their opportunity sets to the same percentage for firms in the largest group.

I set group 8 to have the demand slope from Table I. The supply and demand slopes are then scaled appropriately across the rows and columns of Table II. To cover firms that have even smaller investor bases than in Merton's sample (e.g. small private firms), I add two additional rows with investor bases 10 and 100 times smaller than group 1. To capture a range of supply elasticity values, I include columns with supply elasticities ranging from 1000 times less than the corresponding estimate in Table I to 10 times higher. To illustrate the likely possibility that firms with smaller

Demand slope (Δ_D)	0.666	
	(0.143)	
Supply slope (Δ_S)	-73.635	
	(42.129)	
Active share	1.00	0.35
Investor contribution multiplier $\mathcal{C}^I$ (%)	0.9	2.6

Table I. **Investor contribution multipliers based on homogeneous slope estimates**. This table reports estimates related to the investor contribution multiplier for the large- and medium-sized U.S. stocks. The upper panel reports the slope estimates from Betermier et al. (2022). The lower panel reports the investor contribution multipliers that can be inferred from these slopes and either an active share of 1.00 or the active share based on Koijen et al. (2022). The slope estimates are for an unbalanced panel of 1,226 U.S. firms observed on December 31 of years 2004, 2009, and 2014.

investor bases have much less passive investment, I vary the active share from 0.3 for the largest group to 1 for group 1.

Table II illustrates two main points. First, investor contribution multipliers can vary by multiple orders of magnitude depending on the supply and demand elasticities. The strong relationship between firm size and demand elasticity found in Haddad et al. (2021) underscores one side of this variation. Second, the variation need not all be due to demand elasticity. A firm that is associated with relatively inelastic demand but also a low supply elasticity, and which is only covered by active investors, can offer an investor contribution that is significantly lower than that of much larger firms that are associated with more elastic demand. Investors that are systematically able to identify firms with high supply elasticities can potentially generate relatively large investor contributions, even if these firms are not the most neglected in terms of the demand elasticity of their investor base. Similar to the conclusion of Blitz and Swinkels (2020), investors may wish to differentiate their policies depending on the assessed supply and demand elasticities of individual firms, which are likely to vary both in the cross-section and over time.

The research implication of these results is that if investors with impact preferences are accounting for variations in the relevant investor contribution multipliers, both in the cross-section and over time, then this could have an effect on estimation results.

Merton Group	% Total Market Value	Relative Size of Investor Base	AS	Relative Demand Elasticity	Relative Supply Elasticity .001	.01	.1	1	10
		0.0002	1	0.001	0.8	7.6	45.1	89.1	98.8
		0.002	1	0.01	0.1	0.8	7.6	45.1	89.1
1	0.1	0.02	1	0.1	0.0	0.1	0.8	7.6	45.1
2	0.3	0.03	1	0.17	0.0	0.1	0.6	5.8	39.3
3	0.7	0.04	0.6	0.22	0.0	0.1	0.5	4.9	36.4
4	1.2	0.05	0.6	0.28	0.0	0.1	0.5	4.9	38.0
5	2	0.06	0.6	0.33	0.0	0.0	0.5	4.8	39.1
6	3.2	0.09	0.35	0.5	0.0	0.0	0.4	4.0	34.3
7	5.3	0.13	0.35	0.7	0.0	0.0	0.3	3.1	28.0
8	8.7	0.18	0.35	1.0	0.0	0.0	0.3	2.6	23.9
9	15.1	0.34	0.35	1.9	0.0	0.0	0.1	1.5	14.2
10	63.5	1	0.35	5.6	0.0	0.0	0.0	0.5	5.4

Table II. **Potential variation in investor contribution multipliers**. This table reports illustrative investor contribution multipliers for firms with different associated supply and demand elasticities. The ten rows of numbered groups, and the '% Total Market Value' and 'Relative Size of Investor Base' columns are based on Table I in Merton (1987). The groups were created by ranking firms by market value and splitting them into groups with the same number of stocks in each group. The relative size of the investor base is an estimate of the proportion of investors that have the average stock from a group in their opportunity sets. 'AS' is the active share assumed for each group. The investor contribution multipliers are reported as percentages. The table is calibrated so that the cell with relative supply and demand elasticities of 1 has the investor contribution given in Table I. Cells with investor contributions less than this reference value are shaded gray.

IV.B. Impact Returns and Prices of Impact

Impact returns. In the first part of this subsection, I convert empirical estimates of the demand shifts associated with popular sustainability-related metrics into impact returns. These results are summarized in Table III.

Social score. As part of their study, Betermier et al. (2022) estimate that a one-standard deviation improvement in a firm's MSCI 'Social' (S) score reduces its equilibrium Sharpe ratio by 0.025. For the average firm in their cross-section, with a return volatility of 36%, such a decrease corresponds to a 0.9 percentage point absolute decrease in expected return. Their findings suggest that changes in nonpecuniary characteristics can indeed have an economically significant impact on firms.

Given the heterogeneity observed in Koijen et al. (2022), for the overall market to price in an impact return of $IR^{1SD} = 0.9\%$, it must be the case that some investors apply much higher impact returns to the same assets. To illustrate this in a way that is consistent across the metrics examined in this section, I also report results for demand coefficients that are five times larger. Such larger coefficients are still well within the ranges reported by Koijen et al. (2022) and Noh et al. (2023).

Environmental score. Using an approach focused on leveraging investor holdings data, but using a reasonably similar U.S. data set to the above estimates, Koijen et al. (2022) estimate the shift in investor demand curves from a one standard deviation increase in firms' Sustainalytics 'environmental' (E) score. They report estimated demand coefficients for each institution in their sample, with the estimates ranging from a roughly -40% decline in demand to a 25% increase in demand (i.e., percentage increase in the target weight in the institution's portfolio). As a representative value I use the average coefficient of roughly 5% for households (the largest class of investor in the sample)[11].

I convert this estimate into an implied impact return and price of impact for the average firm in the cross-section of Betermier et al. (2022). Based on the Sharpe ratio of this average firm, a 5% increase in demand corresponds to a 0.024 change in Sharpe ratio and a 0.9% shift in return. These values are remarkably similar to the same values for the social score.

Emissions intensity and adjusted environmental score. Noh et al. (2023) follow an approach

[11] This is a rough estimate as this result is only reported in a histogram chart, not a table. Passive investors, long-term investors and brokers have lower average demand coefficients, but of the same sign and order of magnitude, ranging from 1% to 4%. Hedge funds, small active investors and private banking have negative coefficients, ranging from -1% to -4%.

similar to Koijen et al. (2022), but with a dataset of U.S. stocks from 2013 to 2021 and with both an environmental score and a greenhouse gas (GHG) emissions intensity score. Their log-emissions intensity is the logarithm of firms' Scope 1 GHG emissions to revenue ratios. The log-emissions intensity has an average cross-sectional correlation of 58% with the MSCI environmental score, so they define their environmental score as the residuals of this score after regression on the log-emissions intensity. I convert the estimated demand coefficients for these scores into impact returns using the same procedure as for Koijen et al. (2022)'s environmental score. The combined impact return for these metrics is similar in value to the S-score and E-score impact returns reported above.

Note that Betermier et al. (2022)'s estimation takes place on five-year windows while Koijen et al. (2022) and Noh et al. (2023) use quarterly data. Intuitively this would be more likely to be an issue if I were comparing price elasticity estimates from these different horizons as both supply and demand are likely to appear more elastic at longer horizons (Gabaix and Koijen, 2021; Van der Beck, 2022). However, it is not clear that there should be a similar horizon-dependence problem for impact returns and the similarity of the results from the different sources in Table III offers some comfort in this regard.

Prices of impact. Next, I infer the prices of impact associated with the impact returns reported in Table III. For compatibility with how I treat the emissions intensities (which are in tons of GHG emissions per million dollars revenue), I assume that the sustainability-related good relevant to the sustainability-related scores is the sum of firms' revenues multiplied by their respective scores. The price of impact for a one standard deviation improvement, γ^{1SD}, is then the investor's willingness to pay to improve the sustainability-related score associated with one dollar of revenue.

To calculate the γ^{1SD} prices of impact, I use equation (11) with the larger investor contribution multiplier from Table I, $\mathcal{C}^E = 1$ and $g/P = 0.5$. The latter term is based on a market value to revenue ratio of 2, which I take to be the average for the relevant market[12]. This reflects that the impact returns are relative to market values, while as noted above, I assume that the investors' real concern is the operating size of the firms in terms of their revenues.

Finally, for the log-emissions intensity I convert the one standard deviation prices of impact into prices of impact per tonne of GHG emissions averted. The choice of the relevant conversion factor is not straightforward because the metric is the logarithm of the intensity not the actual intensity.

[12]One point of evidence for this ratio on a relevant population is that Fossil Free Funds report that the average GHG emissions intensity per dollar invested of the Russell 1000 is 94, while the revenue intensity is 181. These Russell 1000 carbon footprint values are from https://fossilfreefunds.org/fund/ishares-russell-1000-etf/IWB/carbon-footprint/FSUSA00B5K/FEUSA0001A. Accessed on 2022-10-31.

To be conservative in the direction of a lower GHG price of impact, I assume that the relevant standard deviation factor is equivalent to 1,000 tons of GHG emissions per million dollars revenue. This is larger than the standard deviations of the Scope 1 GHG-emissions intensities reported in Bolton and Kacperczyk (2021) and Noh et al. (2023) which are near 600 (with much lower standard deviations for Scopes 2 and 3). I divide the relevant γ^{1SD} prices by this factor to arrive at the γ^{GHG} prices of impact.

Bottom-up estimates for GHG emissions. As discussed in Section III, it is also possible to form a bottom-up estimate of impact returns based on prices of impact informed by charitable cost-effectiveness estimates and social costs.

What price of impact might investors choose for GHG emissions? Carbon markets and government policies have typically priced emissions at between \$10-100 per ton of GHG emissions in recent years. Rennert et al. (2022) suggests the social costs should be set at \$185 based on their models. However, in the context of optimal policy for the marginal investor, it is important that the world's largest governments have set much lower prices than this and some prominent charity evaluators argue targeted donations can achieve cost-effectiveness better than \$1 per ton[13]. Therefore, I use \$1 per ton and \$100 per ton to define a range of plausible prices of impact based on observed charitable and social costs.

For a standard deviation of 1,000 tons of GHG emissions per million dollars revenue, a price of \$100 per ton implies $\gamma^{1SD} = 0.1$, whereas \$1 per ton implies $\gamma^{1SD} = 0.001$. I then convert these prices into implied impact returns using equation (11) as above. The results are summarized in Table III.

IV.C. Discussion

The empirical results from the previous subsection reveal a tension between the observed sustainability-related tilts and impact preferences. The prices of impact reported in Table III imply that for the observed tilts to be consistent with impact preferences, some investors would need to be willing to pay substantial fractions, or even more than a dollar, in order to improve the sustainability-related score associated with one dollar of revenue. This suggests an implausible level of latent investor commitment to subsidizing more sustainable firms. The observed prices of impact are also all much larger than the bottom-up prices of impact associated with GHG emissions based on observed estimates of social costs and charitable cost-effectiveness.

[13]See, for example, https://founderspledge.com/stories/climate-and-lifestyle-report.

				Prices of impact	
Source	Metric	Demand coefficient	IR^{1SD}	γ^{1SD}	γ^{GHG}
BCJ	S score	0.025	0.9%	71%	
BCJ	S score	0.125	4.5%	35%	
KRY	E score	0.05	0.9%	67%	
KRY	E score	0.25	4.3%	333%	
NOS	log GHG	-0.023	-0.4%	31%	306
NOS	log GHG	-0.115	-2.0%	153%	1,530
NOS	Residual E	0.031	0.5%	41%	
NOS	Residual E	0.155	2.6%	206%	
	GHG		-0.001%	0.1%	1
	GHG		-0.1%	10%	100

Table III. **Impact returns and prices of impact**. This table reports estimates of the impact returns and prices of impact associated with the findings of Betermier et al. (2022) (BCJ), Koijen et al. (2022) (KRY), and Noh et al. (2023) (NOS), and bottom-up estimates for GHG emissions. The results are shown both for the original sample-average demand coefficients and values five times larger to illustrate the range of the demand coefficients. The impact returns are all for a one standard deviation increase in the relevant metric. The prices of impact are the price to improve the social productivity of one dollar of revenue by one standard deviation and to avert 1 ton of GHG emissions, respectively. The GHG estimates are shown for low and high prices of impact, based on published social costs and charitable cost-effectiveness estimates.

This tension is further underscored by the fact that the bottom-up prices of impact are high enough to justify divestment in some cases, even though my model generally aligns with Berk and Van Binsbergen (2021)'s argument that the impact of divestment is limited. To elaborate, for an investor to divest a firm in my model (in particular, the specification of Section II) requires that their impact tilt offset the small risk premium associated with the firm's correlation with itself as part of the market portfolio (the "internal market correlation" in the terminology of Betermier et al. (2022)). The average internal market correlation reported by Betermier et al. (2022) is 15 basis points. Assuming short selling constraints, this small value is the lowest impact return an investor can meaningfully assign to the average divested firm—whereas positive investments in high impact firms can be generated by impact returns with no upper bound. The investor contribution multiplier reported in Table I means that investors who do divest will have an even smaller effect on the underlying firm's size than the already small impact returns associated with divestment (i.e., 15 bps) would suggest and 15 basis points is above the range for the bottom-up GHG impact returns, IR^{1SD}, reported in Table III. Nevertheless, investors with a higher price of impact may still coherently divest from firms that have emissions intensities that are significantly above average, despite the limited investor impact.

There are many possible resolutions to the tension between observed tilts and impact preferences. First, the investors in the relevant data may simply not have a taste for investor impact in the way I have defined it in this paper. In line with this possibility, Heeb et al. (2022) find that investors have a substantial willingness to pay for climate impact, but that they are not scope sensitive. Such investors may have a taste for impact that is not about their investor impact, but rather about their reputation or other qualitative benefits that depend on their holdings. Indeed, ESG ratings, by boiling complex quantitative criteria that vary dramatically across firms into simple linear scores, may be catering (or contributing) to such scope-insensitivity.

For other investors, such as professional impact activists, it could be that their "theory of change" is not well captured by my model (which is focused on the impact of capital allocation) or by simple sustainability-related scores. They might have an explicit model of their impact that finds higher potential in firms that appear more susceptible to activist campaigns, that have sustainability-related ratings momentum, or that are near a subjective threshold between good and bad sustainability-related performance. It could also be that first-order approximations oversimplify important parts of many firms' effects on the sustainability-related good. For example, a firm's marginal project could be much dirtier than its average project, so that reducing its size at the margin may be much more impactful than my baseline model would indicate. Investors could also have a taste for investor impact as I have defined it, but believe that their investor contribution is higher than implied by my model. For example, I have not allowed for investor's demand shifts to have persistent effects beyond the single period in my model.

Second, investors could believe that higher sustainability-related scores are predictive of future profitability or other financial benefits (such as lower cash flow volatility). This would be captured by holdings-based preferences, taking contribution multipliers out of the picture and thereby dissolving the tension. However, attempts to advocate for this resolution would need to square this result with the increasing number of papers that find investors are willing to pay for impact, including Heeb et al. (2022) and Barber et al. (2021).

Third, it could be that the empirical results I have used as a basis for my analysis are somehow inappropriate for the role I have given them. That is, if investor contribution multipliers are positively correlated with the impact return applied by the market to each firm, then the estimates of Betermier et al. (2022) may be valid on their own terms, but biased when it comes to estimating the (investor-contribution adjusted) price of impact.

Note that there are factors I have not included in this analysis that could widen the gap between the observed and bottom-up prices of impact. In particular, I have not adjusted for enterprise contribution multipliers or the more complex terms in equation (6). I expect these terms would result in a small downwards adjustment to the contribution multiplier for an average firm, which would increase my estimates of the observed prices of impact.

In general, treating supply and demand in an integrated way has important implications for empirical work. At longer horizons, it could be that cases of high real impact due to elastic supply also feature little price impact, while situations of low real impact due to inelastic supply feature the price impacts that have been the target of much empirical work.

V. Extensions

In this section I consider extensions to my baseline setting to allow firms to endogenously modify their social productivities and to allow for a search model of investment. These extensions are important because they address common responses to the direct form of investor impact studied in this paper: i) that what matters is not changing firm size, but changing firmm social productivities, and ii) that the contribution multiplier model, though abstract and general, does not apply to private markets.

V.A. Endogenous Social Productivities

Firm parameters are fixed in my baseline model. As in Pástor et al. (2021), I now defined $G(n) = g(n)K(n)$ and allow firm n's manager to choose $g(n)$. This is particularly relevant to the extensive literature that has focused on the ability of markets to induce firms with low social productivity to reform.

Each firm has initial social productivity $g_0(n)$. The manager chooses $\Delta g(n)$ which moves the firm's total social productivity to $g(n) = g_0(n) + \Delta g(n)$ but with a cost of $-\frac{\chi(n)}{2}(\Delta g(n))^2$ for each unit of installed capital $K(n)$, where $\chi(n) > 0$ controls the adjustment costs.

In Appendix E, I solve for the general case with multiple firms. For the case of a single firm, the result is encapsulated in the following proposition.

Proposition V.1. ***Optimal demand shift with endogenous social productivity***. *The optimal demand curve shift with endogenous social productivities is approximately*

$$\frac{\gamma_i^g}{(1+r_f)}\left(g(n)\mathcal{C}^E(n)\mathcal{C}^I(n)\Big(1+\frac{\Delta g(n)}{g(n)}\Big)+\frac{\zeta_D^{-1}(n,n)K(n)}{\chi(n)g(n)}\right). \tag{31}$$

For the microfounded case of Section II, this shift further simplifies to

$$\frac{\gamma_i^g}{(1+r_f)}g(n)\mathcal{C}^E(n)\mathcal{C}^I(n)\Big(1+\frac{\Delta g(n)}{g(n)}+\frac{\mu(n)}{\chi(n)g(n)^2}\Big), \tag{32}$$

where $\mu(n)$ may be interpreted as the return on assets of the firm.

The terms in parentheses correspond to three channels for investor impact, respectively: increasing the firm size $K(n)$ with $g(n)$ fixed (my baseline channel), increasing firm size via an increase in $\Delta g(n)$, and increasing $\Delta g(n)$ given fixed $K(n)$. The contribution multiplier $\mathcal{C}(n,n)$ applies to *all* of these channels, at least for the microfounded case of Section II. The second channel will be stronger for firms where the endogenous increase in their social productivity ($\Delta g(n)$) is high as a percentage of their social productivity. If $g(n) < 0$ this will only produce a positive investment tilt if $\Delta g(n)/g(n) < -1$. The third channel will be stronger for firms with small costs to improve social productivity ($\chi(n)$), high net productivity ($\mu(n)$) and low social productivity ($g(n)$). However, the results of Berg et al. (2023) are indicative of high costs of improvement for most firms, except in regard to governance processes. Furthermore, if $g(n) < 0$ (or $g(n)\mathcal{C}^E(n) < 0$ in the microfounded case) then this third channel will have a negative contribution to demand.

V.B. Model with Search Frictions

In this subsection, I present a simple model with search frictions to illustrate that contribution multipliers arise in this alternative setting. This relates to Carter et al. (2021)'s call for "probabilistic" approaches to additionality.

Consider a firm in my general baseline setting but suppose that this firm can only take investment from one investor (e.g., a private equity owner). The investor that they meet and receive investment from is randomly drawn at $t = 0$ from among the set $\mathcal{I}_n$ of investors that have agreed to include the firm in their opportunity sets. I assume the selected investor then makes a competitive investment in the firm (i.e., if their bid is not competitive, the firm may solicit a new round of investors), so the formulas for a competitive equilibrium from my baseline model continue to apply.

Conditional on an investor being selected, the difference between the sustainability-related good with and without their investment will be the same as the difference with and without the firm. This is equal to the enterprise impact defined in Appendix A.3. This enterprise impact accounts for the effect of the investment in firm n on other firms. Let $\bar{E}I_i(n)$ be the average enterprise impact associated with firm n, averaged across all investors other than i. The investor impact of investor i choosing to participate in the pool of investors for the firm is then

$$II_i = \frac{1}{|\mathcal{I}_n|}(EI_i(n) - \bar{E}I_i(n)) \tag{33}$$

where $EI_i(n)$ is the enterprise impact of firm n conditional on investor i being the selected investor.

As shown in Appendix A.3, any enterprise impact can be broken down into $EI(n) = g(n)\mathcal{C}^E(n)K(n)$, where in this case $\mathcal{C}^E(n)$ and $K(n)$ depend on which investor is selected via their effect on the overall demand elasticity matrix, as in equation (20. Thus, investor i can produce a positive investor impact by either increasing the size of the firm $K(n)$ or increasing the associated enterprise contribution multiplier. There are many ways such effects could arise. Investor i will invest more in the firm than the average investor if it believes that the firm is more profitable, socially productive or less correlated with the systematic risk in their portfolio. Similarly, the associated enterprise contribution multiplier will tend to be higher if the firm is less correlated with the other assets in the portfolio of the investor (especially with firms with larger values of g). This latter channel highlights how investors that are new to impact investing can add value simply by diversifying the set of investors who are investing in similar firms.

All of the above dimensions of the problem will depend on the supply and demand elasticities associated with the firm and the investors. Furthermore, in relation to my baseline model, the factor of $\frac{1}{|\mathcal{I}_n|}$ is a proxy for the overall demand slope associated with the firm. Thus, elasticity-dependent contribution multipliers also arise in this setting.

If it is not possible for investors to coordinate, then this simple setting can also generated mixed strategies where each investor agrees to include the firm in their opportunity set with some probability. To see how this can arise in the simplest case, suppose all investors have the same beliefs, wealth and existing portfolios, so there will be no difference between the firm receiving investment from any investor. The only way there will be investor impact for investor i is if no other investors included the firm in their opportunity sets so that $\bar{E}I_i(n) = 0$. Suppose there is some cost associated with an investor including the firm in their opportunity set. Then, since all investors have the same beliefs, if they all believe no one else is considering the firm, they will all want to include it in their search (assuming the expected investor impact justifies the cose). Whereas, if they all believe the firm has interest from other investors, so there will be no investor impact, then none of them will want to include it in their search and the firm will not receive investment. This is related to the El Farol bar problem (see, for example, Gintis (2009)). The natural solution is for each investor to choose to include the firm in their opportunity set with some probability such that the expected number of investors looking at the firm is 1. When the number of investors is large, this will generally lead to the firm receiving investment, but with the minimum expected investor search cost (though coordination will always be a way to reduce this cost).

A more detailed search model would allow for search costs for both firms and investors and the possibility of a failure to match. This can affect the decision of firms to embark on a search in the first place. The entrance of a new investor into a market can then affect the rate of successful matching, the volume of matches, the amount of firms that pursue matches and the speed of the matching process. All of these effects are likely to depend on supply and demand-driven contribution multipliers.

VI. Limitations and Potential Extensions

This section discusses the limitations of the model presented in this paper, along with potential extensions and other possibilities for future research. By addressing these limitations and exploring possible extensions, this research can provide a more comprehensive understanding of the effects of impact preferences in financial markets. The model focuses on three key ideas: non-trivial supply elasticities, contribution multipliers, and the price of impact, which together lead to the concept of impact returns. There are numerous natural extensions that could be important in certain contexts.

The model is for a single period, and the microfounded case is for an all-equity economy. Betermier et al. (2022) include debt in their model and, in their appendix, show how to extend the model to multiple periods. As such, I don't expect such extensions to be problematic or to cause any high-level changes to my framework. That is not to say such extensions would not be valuable. In particular, I expect that the price of impact could be made endogenous in a multi-period model. It also seems likely that the naive solutions to these models will be brittle without accounting for model uncertainty and robustness considerations such as those discussed in Hansen and Sargent (2011), Bidder and Dew-Becker (2016), Watson and Holmes (2016), and Yasuda (2023), especially uncertainty about impact and social productivities.

An important impact-specific consideration that could be examined within the framework of this model are nonlinearities associated with the sustainability-related good (e.g., threshold values). For example, adding equity issuance costs could create demand thresholds necessary for firm management to make changes. Similarly if fixed costs of change are added to the setting of Section V.A. It could then be exceptionally high impact for an investor to shift their demand to firms that are near the threshold for change. Limitations to short selling might also create thresholds beyond which demand becomes much less elastic.

Other features that could be more explicitly studied within this framework include: variation in enterprise contribution multipliers, locally increasing returns to scale for selected firms, the "valley of death" phenomena in some markets, firm manager preferences beyond economic profit (e.g., for the sustainability-related good, cash flows), investor utility depending on the wealth of other impact-focused investors, heterogeneous and endogenous investor beliefs about firm profitability and social productivity.

Developing models of blended finance seems like a natural application of this framework, as the real effects of different financial structures will depend on the supply and demand associated with firms and investors in each market. This could include applications to advanced market commitments, social impact bonds, impact certificates and similar. The outcomes for an investor with impact preferences could be contrasted between one scenario where they make impact investments and another where they philanthropically fund payments for the desired outcome—leveraging the capital of other investors who don't have (the same) impact preferences. More complex extensions could be made to cases where firms are able to issue multiple types of securities—this would allow for applications to impact investments with senior and junior (also known as, "catalytic") tranches.

The study of universal ownership also seems like a natural application of this model. Indeed, Quigley (2022) presents a framework for the practice of universal ownership that aligns with my model in several ways. They emphasize a focus on leverage points (where supply elasticities are particularly high)—though the heterogeneity that I emphasize with my results suggests relative optimism about the potential for some investors to generate asset allocation impact in listed equities. They argue traditional sustainable investing aims to protect individual portfolios from systemic risks, while universal owners' interest is in actually mitigating these risks—just like the investors in my model (i.e., if the sustainability-related good is a reduction in systemic risk). The cost-sensitivity matrix Σ_C of Section II is one way to go beyond standard portfolio theory and capture the systemic effect of firms on each other. Such features make this a natural framework within which to define a portfolio theory that meets the demands of universal ownership theory.

Gardner and Henry (2023) present a "dual-hurdle framework" for infrastructure investments in poor countries based on considering two alternative uses of capital as an opportunity cost. This aligns with my discussion of how investors with impact preferences will optimally set their price of impact based on a minimum upper bound inferred from their opportunity costs. Adding contribution multipliers to their framework could be an interesting direction for future research.

While the model does not include an engagement channel, the results suggest new directions for research in this area. For example, do investors target firms that are deemed to have higher enterprise contributions or to offer higher investor contribution multipliers? Do investors strategically target their engagement efforts towards firms that are "under supplied" with engagement, thereby generating a higher investor (engagement) contribution?

There are also important considerations related to impact preferences that are not addressed in the model. First, there is "impact risk", which arises if firms' social productivities are uncertain[14].

[14]While I interpret risk to mean the volatility of a firm's contribution to the sustainability-related good, I have observed that many impact investors use 'impact risk' to mean the probability that a firm's contribution will be

While variation of a firm's social productivity is a practical reality, I am not aware of data that is suitable to study how investors respond to impact risk (though Avramov et al. (2021a)'s approach of using disagreement could be a useful proxy). Additionally, as discussed in Harris (2021a), there are nuanced normative reasons based on moral philosophy to expect that impact risk is a secondary priority for investors with impact preferences. At the very least, how to treat impact risk is a non-trivial topic. Second, investors may assign higher value to investments with returns that are negatively correlated with their future price of impact, a concept first introduced as "mission hedging" in Roth Tran (2019), and that in general may be called "mission correlation" (Harris, 2021a; Harris, 2021b) . Baker et al. (2022b) focus on hedging climate risk outcomes, in a way that seems distinct from, but similar to mission correlation.

VII. Conclusion

This paper presents a model of financial markets as a supply and demand system, with a sustainability-related good, in order to examine the effects of impact preferences. The impact of investor demand shifts, and the optimization of these shifts, hinges on the aggregate supply and demand elasticities through "contribution multipliers". Firms characterized by higher supply elasticities and lower demand elasticities offer greater contribution multipliers, making them more attractive opportunities for investing for impact.

Drawing on empirical findings in the literature, I demonstrate that the observed impact returns and prices of impact associated with prominent sustainability-related metrics are strikingly high, substantially surpassing values suggested by known social costs and charitable cost-effectiveness estimates. Additionally, I reveal that the cross-sectional variation in contribution multipliers can range across orders of magnitude. These results highlight several avenues for future research.

In summary, this paper emphasizes the importance of integrating the supply side into asset pricing models, particularly when exploring real effects such as investor impact. Advancing this direction of research offers a substantial opportunity for empirical and theoretical contributions. Moreover, such research has considerable implications for investors seeking to enhance their investor impact, as well as for regulators and policymakers aiming to effectively oversee these efforts.

realized. In my model this probability is already factored into $g(n)$.

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A. Investor Impact

A.1. Proof of proposition I.1

When $\boldsymbol{K}_i$ changes to $\boldsymbol{K}_i' = \boldsymbol{K}_i + \Delta\boldsymbol{K}_i$, market clearing is maintained by a change in prices. Differentiating both sides of the market clearing condition with respect to $\Delta\boldsymbol{K}_i$ yields

$$\boldsymbol{I} + \sum_{i=1}^{I} \frac{\partial \boldsymbol{K}_j}{\partial \boldsymbol{P}} \frac{\partial \boldsymbol{P}}{\partial \Delta \boldsymbol{K}_i} = \frac{\partial \boldsymbol{K}_S}{\partial \boldsymbol{P}} \frac{\partial \boldsymbol{P}}{\partial \Delta \boldsymbol{K}_i}. \tag{34}$$

Rearranging this becomes

$$\frac{\partial \boldsymbol{P}}{\partial \Delta \boldsymbol{K}_i} = (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}. \tag{35}$$

This implies that the derivative of $\boldsymbol{K}$ with respect to $\Delta\boldsymbol{K}_i$ is

$$\boldsymbol{\mathcal{C}} \equiv \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{K}_i} = \tilde{\boldsymbol{\zeta}}_S \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{K}_i} = \tilde{\boldsymbol{\zeta}}_S (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}. \tag{36}$$

Thus, to first order, the investor impact of a demand shift $\Delta\boldsymbol{K_i}$ is

$$II_i(n) = \boldsymbol{g}' \Delta \boldsymbol{K} = \boldsymbol{g}' \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{K}_i} \Delta \boldsymbol{K_i} = \boldsymbol{g}' \boldsymbol{\mathcal{C}} \Delta \boldsymbol{K_i}. \tag{37}$$

A.2. Inverse matrix decomposition

Suppose we would like to understand the investor impact of a change in demand for firm n, $\Delta K_i(n)$. Without loss of generality, assume $n = 1$ (i.e., is the first element in $\boldsymbol{K}$). Let $\tilde{\boldsymbol{\mathcal{M}}}_D = \tilde{\boldsymbol{\zeta}}_D^{-1}$, $\tilde{\boldsymbol{\mathcal{M}}}_S = \tilde{\boldsymbol{\zeta}}_S^{-1}$, $\tilde{\boldsymbol{\mathcal{M}}}_T = \tilde{\boldsymbol{\mathcal{M}}}_D + \tilde{\boldsymbol{\mathcal{M}}}_S$ and $\boldsymbol{\beta}_{M,n} = (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1} \tilde{\boldsymbol{\mathcal{M}}}_T(-n,n)$. The inverse of $\tilde{\boldsymbol{\mathcal{M}}}_T = \tilde{\boldsymbol{\mathcal{M}}}_D + \tilde{\boldsymbol{\mathcal{M}}}_S$ may be decomposed using the following analytic blockwise inversion formula:

$$\tilde{\boldsymbol{\mathcal{M}}}_T^{-1} = \begin{pmatrix} \tilde{\boldsymbol{\mathcal{M}}}_T(n,n) & \tilde{\boldsymbol{\mathcal{M}}}_T(-n,n) \\ \tilde{\boldsymbol{\mathcal{M}}}_T(n,-n) & \tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n) \end{pmatrix}^{-1} \tag{38}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1} \end{pmatrix} + \begin{pmatrix} 1 \\ -\boldsymbol{\beta}_{T,n} \end{pmatrix} (\tilde{\boldsymbol{\mathcal{M}}}_T(n,n) - \tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)\boldsymbol{\beta}_{T,n})^{-1} \begin{pmatrix} 1 \\ -\tilde{\boldsymbol{\beta}}_{T,n} \end{pmatrix}',$$

with

$$\boldsymbol{\beta}_{T,n} = (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1}\tilde{\boldsymbol{\mathcal{M}}}_T(-n,n), \tag{39}$$

$$\tilde{\boldsymbol{\beta}}_{T,n} = (\tilde{\boldsymbol{\mathcal{M}}}_T(-n,-n))^{-1}\tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)', \tag{40}$$

where subscripts with a single n represent single rows and columns, and subscripts with $(-n)$ refer to all rows or columns excluding n.

A.3. Enterprise impact

Suppose that the aggregate supply and demand curves are $\boldsymbol{P} = \boldsymbol{\mu}_S + \boldsymbol{\mathcal{M}}_S\boldsymbol{K}$ and $\boldsymbol{P} = \boldsymbol{\mu}_D - \boldsymbol{\mathcal{M}}_D\boldsymbol{K}$. Let $\boldsymbol{\mu}_T \equiv \boldsymbol{\mu}_D - \boldsymbol{\mu}_S$. In equilibrium the capital amounts will be $\boldsymbol{K} = \boldsymbol{\mathcal{M}}_T^{-1}\boldsymbol{\mu}_T$, so that

$$K(n) = (\mathcal{M}_T(n,n) - \mathcal{M}_T(n,-n)\boldsymbol{\beta}_{T,n})^{-1} \begin{pmatrix} 1 \\ -\tilde{\boldsymbol{\beta}}_{T,n} \end{pmatrix}' \boldsymbol{\mu}_T \tag{41}$$

Define the Enterprise Impact (EI) of a firm's existence as the difference in G between an economy with and without firm n. Assuming no changes to the demand and supply curves associated with the other firms (after the removal of the firm in question), the Enterprise Impact is

$$EI(n) = \boldsymbol{g}'\boldsymbol{\mathcal{M}}_T^{-1}\boldsymbol{\mu}_T - \boldsymbol{g}'_{(-n)}\boldsymbol{\mathcal{M}}_T(-n,-n)^{-1}\boldsymbol{\mu}_T(-n) \tag{42}$$

$$= \begin{pmatrix} g(n) \\ \boldsymbol{g}_{(-n)} \end{pmatrix}' \begin{pmatrix} 1 \\ -\boldsymbol{\beta}_{T,n} \end{pmatrix} (\mathcal{M}_T(n,n) - \mathcal{M}_T(n,-n)\boldsymbol{\beta}_{T,n})^{-1} \begin{pmatrix} 1 \\ -\tilde{\boldsymbol{\beta}}_{T,n} \end{pmatrix}' \boldsymbol{\mu}_T \tag{43}$$

$$= g(n)\mathcal{C}^E(n)K(n), \tag{44}$$

with

$$\mathcal{C}^E(n) = \left(1 - \frac{\boldsymbol{g}'_{(-n)}}{g(n)}\boldsymbol{\beta}_{T,n}\right), \tag{45}$$

the ‘enterprise contribution multiplier’.

A.4. Investor impact

Applying the above inverse matrix decomposition to $\boldsymbol{\mathcal{C}} = \tilde{\boldsymbol{\mathcal{M}}}_T^{-1}\tilde{\boldsymbol{\mathcal{M}}}_D$ to get the change $\Delta K(n)$ caused by a change $\Delta K_i(n)$ yields

$$\begin{aligned}
\Delta K(n) &= \begin{pmatrix}1\\0\end{pmatrix}' \boldsymbol{\mathcal{C}} \begin{pmatrix}\Delta K_i(n)\\0\end{pmatrix} \\
&= \begin{pmatrix}1\\-\tilde{\boldsymbol{\beta}}_{T,n}\end{pmatrix}' \tilde{\boldsymbol{\mathcal{M}}}_D(\cdot,n)\frac{\Delta K_i(n)}{(\tilde{\boldsymbol{\mathcal{M}}}_T(n,n) - \tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)\boldsymbol{\beta}_{T,n})} \\
&= \mathcal{C}^I(n)\Delta K_i(n)
\end{aligned} \tag{46}$$

with “investor contribution multiplier”,

$$\mathcal{C}^I(n) = \mathcal{C}(n,n) = \frac{\tilde{\mathcal{M}}_D(n,n)\Big(1 - \frac{\tilde{\boldsymbol{\mathcal{M}}}_D(n,-n)}{\tilde{\mathcal{M}}_D(n,n)}\boldsymbol{\beta}_{T',n}\Big)}{\tilde{\mathcal{M}}_T(n,n)\Big(1 - \frac{\tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)}{\tilde{\mathcal{M}}_T(n,n)}\boldsymbol{\beta}_{T,n}\Big)}. \tag{47}$$

Similarly, for investor impact,

$$\begin{aligned}
II_i &= \boldsymbol{g}'\boldsymbol{\mathcal{C}}\Delta\boldsymbol{K}_i \\
&= \sum_{n=1}^{N} \begin{pmatrix}g_i(n)\\\boldsymbol{g}_i(-n)\end{pmatrix}' \begin{pmatrix}1\\-\boldsymbol{\beta}_{T,n}\end{pmatrix} \begin{pmatrix}1\\-\tilde{\boldsymbol{\beta}}_{T,n}\end{pmatrix}' \tilde{\boldsymbol{\mathcal{M}}}_D(\cdot,n)\frac{\Delta K_i(n)}{(\tilde{\boldsymbol{\mathcal{M}}}_T(n,n) - \tilde{\boldsymbol{\mathcal{M}}}_T(n,-n)\boldsymbol{\beta}_{T,n})} \\
&= \sum_{n=1}^{N} g_i(n)\mathcal{C}_i^E(n)\mathcal{C}^I(n)\Delta K_i(n).
\end{aligned} \tag{48}$$

B. General Solution

To first order the effect of a change $\Delta \boldsymbol{K}_i$ on $U_i(\boldsymbol{K}_i, \boldsymbol{P}, \boldsymbol{K})$ is

$$\Delta U_i \approx \left(\frac{\partial U_i}{\partial \boldsymbol{K}_i} + \frac{\partial U_i}{\partial \boldsymbol{P}} \frac{\partial \boldsymbol{P}}{\partial \Delta \boldsymbol{K}_i} + \frac{\partial U_i}{\partial \boldsymbol{K}} \frac{\partial \boldsymbol{K}}{\partial \Delta \boldsymbol{K}_i} \right) \Delta \boldsymbol{K}_i \tag{49}$$

$$= \left(\frac{\partial U_i}{\partial \boldsymbol{K}_i} + \frac{\partial U_i}{\partial \boldsymbol{P}} (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1} + \frac{\partial U_i}{\partial \boldsymbol{K}} \boldsymbol{\mathcal{C}} \right) \Delta \boldsymbol{K}_i \tag{50}$$

The first order condition comes from setting the term in parentheses to zero (and taking the transpose keeps the result in my default column orientation):

$$\left(\frac{\partial U_i}{\partial \boldsymbol{K}_i} + \frac{\partial U_i}{\partial \boldsymbol{P}} (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1} + \frac{\partial U_i}{\partial \boldsymbol{K}} \boldsymbol{\mathcal{C}} \right)' = 0 \tag{51}$$

For $U_i = U_i^H(\boldsymbol{K_i}) + U_i^G(\boldsymbol{K})$ the partial derivatives are

$$\frac{\partial U_i}{\partial \boldsymbol{K_i}} = \boldsymbol{h}_i', \tag{52}$$

$$\frac{\partial U_i}{\partial \boldsymbol{P}} = 0, \tag{53}$$

$$\frac{\partial U_i}{\partial \boldsymbol{K}} = \gamma_i^g \boldsymbol{g}_i'. \tag{54}$$

Plugging the above partial derivatives, as well as those from A.1, into the first order condition, rearranging, and accounting for the original demand curves $\boldsymbol{P}_{i,0}$ yields

$$\boldsymbol{P} = \boldsymbol{P}_{i,0}(\boldsymbol{K}_i, \boldsymbol{K}, \boldsymbol{X}) + \frac{1}{1 + r_f} \left(\boldsymbol{h}_i + \gamma_i^g \boldsymbol{\mathcal{C}}' \boldsymbol{g}_i \right). \tag{55}$$

C. Microfounded solution

C.1. Investor demand curves

In this subsection all vectors and matrices should be understood to apply only over the investor's opportunity set $\mathcal{N}_i$.

Investor i's financial wealth at $t = 1$ is

$$W_{i,1} = W_{i,0}(1 + r_f) + \boldsymbol{z}'_{CF} \operatorname{diag}(\boldsymbol{P})\boldsymbol{K}_i - (1 + r_f)\boldsymbol{P}'(\boldsymbol{K}_i - \boldsymbol{K}_{i,0}). \tag{56}$$

The part of each investor's utility that is relevant to the solution of the model is,

$$\tilde{U}_i = \boldsymbol{a}'_i\boldsymbol{K}_i - (1 + r_f)\boldsymbol{P}'(\boldsymbol{K}_i - \boldsymbol{K}_{0,i}) - \frac{\gamma_i}{2W_{i,0}}\boldsymbol{K}'_i\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i + \gamma^g_i\boldsymbol{g}'_i\boldsymbol{K}. \tag{57}$$

The relevant partial derivatives of the utility function are:

$$\frac{\partial U_i}{\partial \boldsymbol{K_i}} = \Big(\boldsymbol{a}_i - (1 + r_f)\boldsymbol{P} - \frac{\gamma_i}{W_{i,0}}\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i\Big)', \tag{58}$$

$$\frac{\partial U_i}{\partial \boldsymbol{P}} = -(1 + r_f)(\boldsymbol{K}_i - \boldsymbol{K}_{0,i})', \tag{59}$$

$$\frac{\partial U_i}{\partial \boldsymbol{K}} = \gamma^g_i\boldsymbol{g}'_i. \tag{60}$$

Here I have ignored that $W_{0,i}$ is endogenous as accounting for this only produces second-order terms (see discussion in Subsection C.4 below).

The first order condition for the demand curves is thus

$$\boldsymbol{a}_i - (1 + r_f)\boldsymbol{P} - \frac{\gamma_i}{W_{i,0}}\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i - (1 + r_f)(\boldsymbol{K}_i - \boldsymbol{K}_{0,i})(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1} + \gamma^g_i\boldsymbol{\mathcal{C}}'\boldsymbol{g}_i = 0. \tag{61}$$

Rearranging, and noting that for an infinitesimal investor $(\boldsymbol{K}_i - \boldsymbol{K}_{0,i}) \to 0$ while $\boldsymbol{K_i}/W_{0,i}$ does not, this implies the optimal demand curve for the investor is

$$\boldsymbol{P} = \frac{1}{1 + r_f}\left(\boldsymbol{a}_i - \frac{\gamma_i}{W_{i,0}}\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i + \gamma^g_i\boldsymbol{\mathcal{C}}'\boldsymbol{g}_i\right), \tag{62}$$

and that the investor's demand elasticity matrix is

$$\boldsymbol{\zeta}_i = (1 + r_f)\frac{W_{i,0}}{\gamma_i}\boldsymbol{\Sigma}_{CF,i}^{-1}. \tag{63}$$

The aggregate demand elasticity is obtained by summing over the individual demand elasticity matrices, taking care to adapt them to the full opportunity set. Let $\boldsymbol{I}_i$ be as defined in the main text. That is, there is a column for each firm in the economy, a row for each firm n in $\mathcal{N}_i$, and each such row is all zeroes except for a single 1 in the column corresponding to its respective n. Using these indicator matrices the result is given by

$$\boldsymbol{\zeta}_D = \sum_{i=1}^{I} \boldsymbol{I}_i' \boldsymbol{\zeta}_i \boldsymbol{I}_i. \tag{64}$$

C.2. Firm supply curves

The partial derivative of firm n's utility with respect to $\Delta K(n)$ is

$$\frac{\partial U_F(n)}{\partial \Delta K(n)} = -\tilde{\boldsymbol{\zeta}}_D^{-1}(n,n)K(n) + P(n) - (1 + \boldsymbol{\Sigma}_C(n,\cdot)\Delta\boldsymbol{K}). \tag{65}$$

Combining all such derivatives into a vector and setting it to zero implies that

$$\boldsymbol{P} = \mathbf{1} + \text{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1})\boldsymbol{K} + \boldsymbol{\Sigma}_C\Delta\boldsymbol{K} \tag{66}$$

$$= \mathbf{1} + \text{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1})\boldsymbol{K}_0 + (\text{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1}) + \boldsymbol{\Sigma}_C)\Delta\boldsymbol{K}. \tag{67}$$

This implies $\tilde{\boldsymbol{\zeta}}_S = (\text{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1}) + \boldsymbol{\Sigma}_C)^{-1}$. Assuming that capital installation costs dominate the aggregate investor demand elasticity (i.e., $\boldsymbol{\Sigma}_C >>> \text{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1})$), then the this simplifies to $\tilde{\boldsymbol{\zeta}}_S = \boldsymbol{\Sigma}_C^{-1}$ so that the supply curves are

$$\boldsymbol{P} = \mathbf{1} + \text{diag}(\tilde{\boldsymbol{\zeta}}_D^{-1})\boldsymbol{K}_0 + \boldsymbol{\Sigma}_C\Delta\boldsymbol{K} \tag{68}$$

$$= \mathbf{1} - \boldsymbol{\Sigma}_C\boldsymbol{K}_0 + \boldsymbol{\Sigma}_C\boldsymbol{K}. \tag{69}$$

C.3. Equilibrium values

Let the aggregate demand curve be written

$$\boldsymbol{P} = \frac{1}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) - \tilde{\boldsymbol{\zeta}}_D^{-1}\boldsymbol{K}, \tag{70}$$

where the vectors and matrices of aggregate parameters are constructed by multiplying the investor level parameters by $(\boldsymbol{I}_i)'(\boldsymbol{I}_i\boldsymbol{\zeta}_i(\boldsymbol{I}_i)')^{-1}\boldsymbol{I}_i$, aggregating and reversing the multiplication.

Solving for the equilibrium based on the supply and demand curves, the firm sizes are

$$\boldsymbol{K} = (\tilde{\boldsymbol{\zeta}}_D^{-1} + \tilde{\boldsymbol{\zeta}}_S^{-1})^{-1}\left(\frac{1}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) - \boldsymbol{1} - \tilde{\boldsymbol{\zeta}}_S^{-1}\boldsymbol{K}_0\right) = (\tilde{\boldsymbol{\zeta}}_D^{-1} + \tilde{\boldsymbol{\zeta}}_S^{-1})^{-1}\boldsymbol{\mu} \tag{71}$$

with $\boldsymbol{\mu} = \frac{1}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) - \boldsymbol{1} - \tilde{\boldsymbol{\zeta}}_S^{-1}\boldsymbol{K}_0$.

Equilibrium prices are

$$\boldsymbol{P} = (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}\left(\frac{\tilde{\boldsymbol{\zeta}}_D}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) + \tilde{\boldsymbol{\zeta}}_S\boldsymbol{1} + \boldsymbol{K}_0\right), \tag{72}$$

$$\approx (\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}\left(\frac{\tilde{\boldsymbol{\zeta}}_D}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) + \boldsymbol{K}_0\right) \tag{73}$$

Assuming the supply elasticities are much smaller than the demand elasticities, and dropping $\boldsymbol{K}_0$ to focus on the endogenous effects results in

$$\boldsymbol{P} \approx \frac{(\tilde{\boldsymbol{\zeta}}_D + \tilde{\boldsymbol{\zeta}}_S)^{-1}\tilde{\boldsymbol{\zeta}}_D}{1+r_f}(\boldsymbol{a} + \gamma^g \boldsymbol{\mathcal{C}}'\boldsymbol{g}) \tag{74}$$

C.4. Effect of endogenous investor wealth

Accounting for $W_{i,0} = \boldsymbol{P}'\boldsymbol{K}_{i,0} + C_i$ the corresponding vector of derivatives with respect to the price shift $\frac{\partial dU_i}{\partial d\boldsymbol{P}}$ to investor i's utility resulting from the shift then includes additional terms $(1 + r_f + \frac{\gamma_i}{2W_{i,0}^2}(\boldsymbol{K}_i'\boldsymbol{\Sigma}_{CF,i}\boldsymbol{K}_i))\boldsymbol{K}_{i,0}$. Assuming the investor was and continues to be approximately diversified across the N firms, this additional term will be small for small $W_{i,0}$ as $\boldsymbol{K}_{i,0} \to 0$. Hence, while the investor's initial share holdings could affect their behavior, this effect should be insignificant in the baseline case with infinitesimal investors.

D. Security Impact Line

In general, consider that the optimal microfounded demand curves of equation (17) may be rewritten as

$$\boldsymbol{EFR}_i + \boldsymbol{IR}_i = \mathrm{diag}(\boldsymbol{\zeta}_i^{-1})\boldsymbol{K}_i. \tag{75}$$

The Security Impact Line follows from setting $\boldsymbol{K}_i = 0$.

E. Endogenous Social Productivities

To solve the model in this case I assume that each investor's demand curve may be written as

$$\boldsymbol{P} = \boldsymbol{P}_{i,0} - \frac{\mathrm{diag}(\boldsymbol{\chi})}{2}\,\mathrm{diag}(\Delta\boldsymbol{g})\Delta\boldsymbol{g} + \gamma_i^g \boldsymbol{\mathcal{M}}_i^g \boldsymbol{g} \tag{76}$$

and, thus, that the aggregate demand curve is

$$\boldsymbol{P} = \boldsymbol{P}_{i,0} - \frac{\mathrm{diag}(\boldsymbol{\chi})}{2}\,\mathrm{diag}(\Delta\boldsymbol{g})\Delta\boldsymbol{g} + \boldsymbol{\mathcal{M}}^g \boldsymbol{g} \tag{77}$$

where $\boldsymbol{\mathcal{M}}_i^g$ is a coefficient matrix to be optimized and $\boldsymbol{\mathcal{M}}^g = \boldsymbol{\mathcal{M}}_D \sum_{i=1}^I \gamma_i^g \boldsymbol{\zeta}_i \boldsymbol{\mathcal{M}}_i^g$. As in Section I.B, I assume that $\boldsymbol{P}_{i,0}$ are the optimal demand curves associated with the investor's pure financial utility ($U_{i,0}(\boldsymbol{K_i}) = U_i^F(\boldsymbol{K_i}) - (1+r_f)\boldsymbol{P'K_i}$) and I derive the optimal $\boldsymbol{\mathcal{M}}_i^g$ given the additional utility term $U_i^G(\boldsymbol{K})$.

First I solve for the firm's policy. Taking the investor demand curves as fixed (though dependent on $\boldsymbol{g}$), each firm n chooses $K(n)$ and $g(n)$ to maximize its economic profit. The optimization with respect to $K(n)$ is as in Appendix C.2. With respect to $g(n)$ the optimization is simply about maximizing the equilibrium price $P(n)$, so that for each firm the first order condition is $\frac{\partial P(n)}{\partial \Delta g(n)} = -\chi(n)\Delta g(n) + \mathcal{M}^g(n,n) = 0$. Stacking together the solutions for each firm this means

$$\Delta\boldsymbol{g} = \mathrm{diag}(\boldsymbol{\chi})^{-1}\,\mathrm{diag}(\boldsymbol{\mathcal{M}}^g). \tag{78}$$

Now consider the first order condition for investor i's utility with respect to the term $\mathcal{M}_i^g(n,m)$.

The partial derivatives of the core variables with respect to this term are as follows:

$$\frac{\partial \boldsymbol{g}}{\partial \mathcal{M}_i^g(n,m)} = \gamma_i^g \begin{pmatrix} \mathbf{0}_{(m-1)\times 1} \\ (\boldsymbol{\mathcal{M}}_D \boldsymbol{\zeta}_i)(m,n)/\chi(m) \\ \mathbf{0}_{(N-m)\times 1} \end{pmatrix} \tag{79}$$

$$\frac{\partial \boldsymbol{K}_i}{\partial \mathcal{M}_i^g(n,m)} \approx \gamma_i^g \boldsymbol{\zeta}_i(\cdot,n) g(m) \tag{80}$$

$$\frac{\partial \boldsymbol{K}}{\partial \mathcal{M}_i^g(n,m)} = \boldsymbol{\mathcal{C}} \left(\boldsymbol{\zeta}_D \boldsymbol{\mathcal{M}}^g \frac{\partial \boldsymbol{g}}{\partial \mathcal{M}_i^g(n,m)} + \frac{\partial \boldsymbol{K}_i}{\partial \mathcal{M}_i^g(n,m)} \right) \tag{81}$$

$$= \gamma_i^g \boldsymbol{\mathcal{C}} \left(\boldsymbol{\zeta}_D \boldsymbol{\mathcal{M}}^g \begin{pmatrix} \mathbf{0}_{(m-1)\times 1} \\ (\boldsymbol{\mathcal{M}}_D \boldsymbol{\zeta}_i)(m,n)/\chi(m) \\ \mathbf{0}_{(N-m)\times 1} \end{pmatrix} + \boldsymbol{\zeta}_i(\cdot,n) g(m) \right) \tag{82}$$

$\frac{\partial \boldsymbol{P}}{\partial \mathcal{M}_i^g(n,m)}$ is similar in nature to $\frac{\partial \boldsymbol{K}}{\partial \mathcal{M}_i^g(n,m)}$, but is unnecessary in the infinitesimal investor limit due to $\frac{\partial U_i}{\partial \boldsymbol{P}} \approx 0$ as indicated below.

The partials of the utility term are

$$\frac{\partial U_i}{\partial \boldsymbol{g}} = \gamma_i^g \boldsymbol{K}, \tag{83}$$

$$\frac{\partial U_i}{\partial \boldsymbol{K}_i} = \frac{\partial U_i^F}{\partial \boldsymbol{K}_i} - (1+r_f)\boldsymbol{P} = -(1+r_f)\gamma_i^g \boldsymbol{\mathcal{M}}_i^g \boldsymbol{g}, \tag{84}$$

$$\frac{\partial U_i}{\partial \boldsymbol{P}} = \boldsymbol{K_i} \approx \mathbf{0}, \tag{85}$$

$$\frac{\partial U_i}{\partial \boldsymbol{K}} = \gamma_i^g \boldsymbol{g}, \tag{86}$$

where equation (84) follows from the assumption that the original demand curves are optimal for the pure financial utility function (i.e., $(1+r_f)\boldsymbol{P}_{i,0} = \frac{\partial U_i^F}{\partial \boldsymbol{K}_i}$).

The resulting first order condition is

$$(\boldsymbol{K}' + \boldsymbol{g}'\boldsymbol{\mathcal{C}}\boldsymbol{\zeta}_D \boldsymbol{\mathcal{M}}^g) \begin{pmatrix} \mathbf{0}_{(m-1)\times 1} \\ (\boldsymbol{\mathcal{M}}_D \boldsymbol{\zeta}_i)(m,n)/\chi(m) \\ \mathbf{0}_{(N-m)\times 1} \end{pmatrix} = \Big((1+r_f)\boldsymbol{g}'(\boldsymbol{\mathcal{M}}_i^g)' - \boldsymbol{g}'\boldsymbol{\mathcal{C}} \Big) \boldsymbol{\zeta}_i(\cdot,n) g(m) \tag{87}$$

The above equation holds for all n, so that

$$\boldsymbol{g}'\boldsymbol{\mathcal{C}} + (\boldsymbol{K}' + \boldsymbol{g}'\boldsymbol{\mathcal{C}}\boldsymbol{\zeta}_D\boldsymbol{\mathcal{M}}^g)\begin{pmatrix} \mathbf{0}_{(m-1)\times N} \\ (\boldsymbol{\mathcal{M}}_D\boldsymbol{\zeta}_i)(m,\cdot)\boldsymbol{\zeta}_i^{-1}/\chi(m)/g(m) \\ \mathbf{0}_{(N-m)\times N} \end{pmatrix} = (1+r_f)\boldsymbol{g}'(\boldsymbol{\mathcal{M}}_i^g)' \tag{88}$$

The above equation holds for all m, so that

$$\boldsymbol{g}'\boldsymbol{\mathcal{C}} + (\boldsymbol{K}' + \boldsymbol{g}'\boldsymbol{\mathcal{C}}\boldsymbol{\zeta}_D\boldsymbol{\mathcal{M}}^g)\operatorname{diag}(\boldsymbol{\chi})^{-1}\operatorname{diag}(\boldsymbol{g})^{-1}\boldsymbol{\mathcal{M}}_D = (1+r_f)\boldsymbol{g}'(\boldsymbol{\mathcal{M}}_i^g)', \tag{89}$$

with solution,

$$\boldsymbol{\mathcal{M}}_i^g = \frac{1}{(1+r_f)}\left(\boldsymbol{\mathcal{C}} + \Big(\operatorname{diag}(\boldsymbol{K})\operatorname{diag}(\boldsymbol{g})^{-1} + \boldsymbol{\mathcal{C}}\boldsymbol{\zeta}_D\boldsymbol{\mathcal{M}}^g\Big)\operatorname{diag}(\boldsymbol{\chi})^{-1}\operatorname{diag}(\boldsymbol{g})^{-1}\boldsymbol{\mathcal{M}}_D\right)'. \tag{90}$$

For a single firm, ignoring cross-elasticities, this simplifies to

$$\mathcal{M}_i^g g \approx \frac{\gamma_i^g}{(1+r_f)}\left(\mathcal{C}^E\mathcal{C}^I\Big(1+\frac{\Delta g}{g}\Big) + \frac{\zeta_D^{-1}K}{\chi g^2}\right)g. \tag{91}$$

Based on Appendix C, ζ_D^{-1} will be approximately $(1+r_f)\frac{\gamma}{W_0}\sigma_{CF}^2$, so it is possible that ζ_D^{-1} is even smaller than K is large. An approximate model of firm size, based on Appendix C, is that $\boldsymbol{K} \approx (\tilde{\boldsymbol{\zeta}}_D^{-1} + \tilde{\boldsymbol{\zeta}}_S^{-1})^{-1}\boldsymbol{\mu}$ where μ is essentially the return on assets of the firm's technology. But $\boldsymbol{\mathcal{C}} = (\tilde{\boldsymbol{\zeta}}_D^{-1} + \tilde{\boldsymbol{\zeta}}_S^{-1})^{-1}\tilde{\boldsymbol{\zeta}}_D^{-1}$. So with this approximation the contribution multiplier applies to every term and may be brought out of the parentheses

$$\mathcal{M}_i^g g \approx \frac{\gamma_i^g}{(1+r_f)} g\mathcal{C}^E\mathcal{C}^I\Big(1+\frac{\Delta g}{g} + \chi^{-1}\frac{\mu}{g^2}\Big). \tag{92}$$